The Exchange Rate Effect of Multi-Currency Risk Arbitrage

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INSEAD and CEPR

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This paper documents how currency speculators trade when international capital flows generate predictable exchange rate movements. The redefinition of the MSCI world equity index in December 2000 provides an ideal natural experiment identifying exogenous capital flows of index tracking equity funds. Currency speculators are shown to front-run international capital flows. Furthermore, they actively manage the portfolio risk of their speculative positions through hedging positions in correlated currencies. The exchange rate effect of separate risk hedging is economically significant and amounts to a return difference of 3.6 percent over a 5 day event window between currencies with high and low risk hedging value. The results of the classical event study analysis are confirmed by a new and more powerful spectral inference isolating the high frequency cospectrum of currency pairs. The evidence supports the idea that international currency arbitrage is limited by the speculators' risk aversion.

JEL classification: G11, G14, G15.

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1 Introduction

This paper investigates how currency speculators trade when international capital flows generate predictable excess returns in multiple currencies. Cross-border capital flows have rapidly increased since the 1990s and have been found to appreciate exchange rates and the reversal of foreign asset positions to cause currency crashes (Brunnermeier, Nagel and Pedersen (2008)). Capital flows into emerging markets are particularly prone to temporal variation which affect simultaneously the exchange rates of multiple currencies (Reinhart and Reinhart (2008)). Predictability of such capital flows gives rise to speculative opportunities in currency market.[1]

The first part of this paper characterizes the optimal arbitrage strategy in a multi-currency setting. It is argued that currency speculation involves risk hedging positions which co-determine cross-sectional exchange returns during arbitrage episodes along with expected excess returns. The second part of the paper documents the quantitative importance of risk hedging based on a unique natural experiment. In December 2000, the most important provider of international equity indices, Morgan Stanley Capital Inc. (MSCI) announced that it would substantially alter the composition of its global equity indices. As a consequence, many countries experienced dramatic changes in their index representation, which resulted in a highly predictable reallocation of indexed equity capital from down- to upweighted currencies. For this exogenous event, the optimal risk arbitrage strategy with its hedging positions is highly predictable and can be compared to the cross-sectional currency return pattern.

Speculative strategies typically require capital, involve risk and use special information (Shleifer and Vishny (1997)). All three aspects have their particular significance in the context of the foreign exchange (FX) market. First, a shortage of arbitrage capital alledgedly characterizes the FX market (Bacchetta and van Wincoop (2008), Jylhä, Suominen and Lyytinen (2008)). This could explain well-known puzzles like the empirical failure of 'uncovered interest parity'. This paper provides new evidence with respect to this debate. Second,

[1] I use the term 'risk arbitrage' and speculation as synonomous in line with the empirical literature on limits of arbitage and much of the investment profession. This terminology is different from the classic theoretical definition which defines arbitrage as a riskless profit opportunity.

the high volatility of exchange rates make risk control very important to speculators. The optimal FX arbitrage strategy should reflect the arbitrageurs' desire to limit risk exposure. Risk control through hedging becomes feasible if the arbitrage strategy involves multiple, correlated currency markets. The multi-currency setting analyzed in this paper can deliver direct evidence for the risk aversion of currency speculators. Third, the opportunity to trade against uninformed 'outsiders' in a very liquid market is also important because it allows for a sufficient return in compensation for speculative risk. In this respect, the FX market with its enormous liquidity is particularly attractive to risk arbitrageurs.

The current paper makes three contributions. First, it develops a dynamic model of limited arbitrage across multiple currencies. The currency market is represented by multiple - stochastically evolving - price elastic net supply functions with a currency-specific supply elasticity. Risk averse arbitrageurs anticipate future capital reallocations and take optimal arbitrage positions across all currencies to front-run these international capital flows.[2] Their arbitrage positions affect the exchange rate return pattern and generate cross-sectional returns with two distinct components: The *fundamental component* is proportional to the expected excess return induced by the capital flows into each currency. Currencies with inflows (outflows) experience an appreciation (depreciation) during the built-up of speculative positions. A second pricing factor, the *risk-hedging component*, is (negatively) proportional to the 'marginal arbitrage risk' of each currency position. Risk averse speculators do not only seek high returns by front-running future capital flows, but also try to minimize the overall risk of their arbitrage positions. As a result, the optimal arbitrage strategy downweights (upweights) currencies which contribute more (less) to the overall arbitrage risk. The fundamental and the risk-hedging effects can occur jointly when the arbitrage strategies are implemented and before the anticipated capital flows occur or become widely known.

A second contribution of the paper is to test these predictions. The MSCI index weight change provides an extremely precise identification for the relative size and direction of future international capital flows. It is conjectured that the corresponding FX arbitrage trading occurred directly prior to MSCI's first public announcement on December 1, 2000. The

[2] For related evidence in equity markets of front-running on mutual fund fire sales, see Chen, Hanson, Hong and Stein (2008) and Coval and Stafford (2007).

cross-section of 37 spot exchange rates exhibit the predicted positive fundamental effect and the negative risk-hedging effect. The explanatory power of the cross-sectional regression is astonishing. Together, the fundamental and the hedging effect account for almost 55 percent of the exchange rate variation over a 3 day window and more than 35 percent over a 7 day window. I confirm these findings for a subsample of the most liquid currencies for which forward rate data is publicly available. Forward rates show the same cross-sectional return pattern as spot rates. The MSCI event reveals that hedging arbitrage risk matters to the currency arbitrageurs, and that this concern is reflected in the risk hedging positions of the arbitrageurs. Such speculative risk hedging has economically significant currency effects. The point estimates suggest that the exchange rate return difference between currencies with high and low hedging benefits (separated by 2 standard deviations in the hedging benefit) amounts to 3.6 percent over the 5 trading days of the event window.

A third contribution is methodological. Conventional cross-sectional event studies lack statistical power if they use less than 40 observations (as is the case for exchange rate studies). A new statistical methodology is therefore proposed which uses high frequency data and inference in the frequency domain to obtain sharper statistical inference. The intuition is as follows: Consider a sequence of speculators, each of whom implements an optimal multi-currency strategy. They will each built their positions simultaneously across all currencies. The price impact across currencies should also be simultaneous and be reflected in the high frequency comovements across different exchange rates.[3] Such high frequency comovements can be measured as the high frequency components of the cospectrum of exchange rate returns. For example, exchange rate pairs for which the arbitrage position is long in both currencies should experience positive high frequency comovements. Instead, exchange rate pairs for which the arbitrage positions are long for one and short for the other should exhibit a more negative covariance at the highest frequencies. The latter corresponds to a negative shift of the high frequency components of the cospectrum.

The high liquidity of the exchange rate market allows the use of minute by minute price data. The cospectrum between a pair of exchange rate returns can be aggregated into a 'very

[3] A common procedure is to filter out 'high frequency' noise as it is strongly determined by trading activity. The current study pursues the opposite objective of identifying particular patterns of cross-currency trading.

high frequency' band summing all comovements within a 15 minute interval, into a 'high frequency' band for comovements from 15 minutes to 1 hour, a 'medium frequency' band capturing comovements between 1 hour and 4 hours and a 'low frequency' band capturing all remaining comovements. It is shown that a large share of the change in the covariance of exchange rate pairs in the 7 day arbitrage period directly prior to MSCI's announcement is due to a change in the very high frequency band of the cospectrum. The event-related change in the exchange rate dynamics is therefore characterized by strong cross-sectional return simultaneity. Moreover, the high frequency cospectrum shift for each currency pair corresponds to the predicted arbitrage positions for the respective currency pair. The event period shift of the very high frequency cospectrum is positive if the speculative positions in both currencies have the same direction (both long or both short). The shift of the very high frequency cospectrum is negative if optimal risk arbitrage requires speculative positions of opposite directions (one short and one long).

Finally, I revisit the model predictions about the fundamental and the risk-hedging components of the arbitrageurs' price impact. Spectral band regressions are used to focus on the very high frequency components of currency comovements. They allow identification of both the fundamental and risk-hedging effects of arbitrage trading at much higher levels of statistical significance than the conventional inference. Here we can also make a quantitative assessment about the role of currency hedging demands on exchange rate returns. The spectral band regressions show that the (transitory) exchange rate effect of the hedging demand is at least as large (and probably larger) than the fundamental effect.

The current paper relates to a large literature on limited arbitrage. This literature is mostly concerned with equity markets with a prominent role played by stock index in- and exclusions.[4] Empirical work on limited arbitrage in the FX market is less common. Most recently, Brunnermeier, Nagel and Pedersen (2008) provide evidence that so-called carry trades alter the distribution of exchange rate movements. The negative skewness of target

[4]See for example Garry and Goetzmann (1986), Harris and Gurel (1986), Shleifer (1986), Shleifer and Vishny (1987), Dillon and Johnson (1991), Beniesh and Whaley (1996), Lynch and Mendenhall (1997). Kaul et al. (2000) examine index reweighting for stocks in the Toronto Stock Exchange 300 index and find that upweighted stocks experience a persistent positive price effect. See also Denis et al. (2003) and Hedge and McDermott (2003).

currencies is interpreted as the result of a sudden unwinding of carry trades. Jylhä, Suominen and Lyytinen (2008) explore the long-run profitability of carry trade strategies and show that the returns to carry trades have been decreasing over the last 32 years. Moreover, carry trade returns explain a significant part of hedge fund index returns. The microeconomics of currency speculation therefore deserves more attention.

An important feature of my analysis is the multi-asset approach to speculation. Such a portfolio approach has recently been employed for speculative equity trading (Greenwood, 2005) and option pricing (Garleanu, Pedersen, and Poteshman, 2008). But in contrast to these papers, the speculators in the current paper face a price elastic residual asset supply. This distinguishing model feature implies that speculators can acquire optimal hedging positions instead of just absorbing an exogenous supply shock. Currency risk arbitrage in this paper amounts to 'net position taking', which brings the model closer to a practitioner's notion of speculation. The model structure is similar to Greenwood and Vayanos (2008), where risk averse speculators choose optimal arbitrage positions against a price elastic net supply in bonds of different maturity. But unlike bond yields in their set-up, exchange rates in my model are governed by asset specific stochastic processes. This implies that the covariance structure of risk becomes an important element determining the optimal arbitrage position.

The paper also relates to a larger literature on capital flows and exchange rate determination. A downward sloping demand curve for foreign exchange balances is related to imperfect international asset substitutability (Kouri (1983), Branson and Henderson (1985)).[5] Empirical work has revealed a strong correlation between capital flows and exchange rate returns. Froot and Ramadorai (2004) for example use a simple VAR framework to document very persistent exchange rate effects related to U.S. institutional in- and outflows. Evidence on the exchange rate impact of order flows is provided by Evans and Lyons (2002). Structural approaches have tried to provide causal inference: Pavlova and Rigobon (2003) and Hau and Rey (2004) use model-based identification assumptions to assess the role of capital flow shocks for exchange rate movements. In the latter studies, causal inference is contingent on

[5] For a survey of the relevant literature, see Rogoff (1984). See also Edison (1993), Payne and Vitale (2003), and Dominquez (2003) for the related issue of central bank intervention in the FX market and its effectiveness.

the validity of the identification assumptions. In contrast, the MSCI event provides a simple model-free identification of exogenous capital flows which generates predictable exchange rate movements as argued by Hau, Peress and Massa (2008).

The following section presents the theory and develops the testable hypotheses for both the spot and forward exchange rate. Section 3 discusses the MSCI index revision, its implications for the country weight changes and the arbitrage risk related to an optimal speculative position. The cross-sectional evidence follows in section 4 for daily spot rate returns and forward rate returns. Section 5 discuses the spectral methodology and the corresponding evidence. Section 6 concludes.

2 Theory and Hypotheses

2.1 Model Assumptions

This section develops a simple limit-to-arbitrage model, in which arbitrageurs take optimal speculative positions in anticipation of an exogenous currency demand shock. The market characteristics are summarized as follows:

Assumption 1: Linear Stochastic Currency Supply

A currency market allows simultaneous trading in currencies $i = 1, 2, 3..., n$. *Trading occurs over the time interval* $[0, T]$ *at (equally spaced) time points* $t = 0, \Delta t, 2\Delta t, 3\Delta t, ...T$, *with* $\Delta t = T/N$. *The (residual) liquidity supply* S_i *of currency* i *is characterized by a linear function of the exchange rate* e_{it} *given by*

$$S_i(e_{it}) = q_i(e_{it} - \Phi_{it} - r_i t), \tag{1}$$

where $q_i > 0$ *is the liquidity supply elasticity of currency* i. *The fundamental values* Φ_{it} *of currency* i *are combined in a stochastic vector* $\Phi_t = (\Phi_{1t}, \Phi_{2t}, ..., \Phi_{nt})'$ *given by*

$$\Phi_{t=k\Delta t} = \mathbf{1} + \sum_{t=\Delta t}^{k\Delta t} \varepsilon_t, \tag{2}$$

in trading round k. *Let* $\mathbf{1}$ *denote a unit vector and innovations* $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t} ..., \varepsilon_{nt})'$ *have zero mean and a covariance* $\mathcal{E}_{t-\Delta t}(\varepsilon_t \varepsilon_t') = \Sigma\Delta t$. *The term* r_i *denotes the one-period money market interest rate in currency* i. *The home money market rate is set to zero.*

Assumption 1 characterizes upward sloping net supply curve for each currency with individual elasticities q_i. It can be motivated empirically as the stylized representation of an aggregate FX limit order book.[6] Currency orderflow moves the currency prices along the liquidity supply schedule and generates persistent exchange rate effects documented in the microstructure literature (Evans and Lyons (2002), Froot and Ramadorai (2004)). In the long-run, the elasticity parameter should be related to the willingness of investors to substitute home assets for foreign assets if the exchange rate becomes more favorable.[7] The linearity of the supply function is chosen for analytical convenience. For the same reason, the supply in each currency depends only on its own price and not on other exchange rates. It is also assumed that all quantities and corresponding elasticities are expressed in the same reference currency. Hence, a multi-currency demand shock denominated in the reference currency and represented by $u = (u_1, u_2, ...u_n)$ changes each exchange rate by $\frac{1}{q_i}u_i$. Since all currencies are initially normalized to 1, it is convenient to refer to the exchange rate change $e_{t+\Delta t} - e_t$ as the (approximate) exchange rate return.[8]

In the absence of any demand shock, market clearing requires $S_i(e_{it}) = 0$ for each time t and each currency i. The equilibrium exchange rate vector $e_t = (e_{1t}, e_{2t}..., e_{nt})'$ follows $e_0 = \mathbf{1}$ for $t = 0$ and for trading rounds numbered $k = 1, 2, ...N$ as

$$e_{k\Delta t} = \mathbf{1} + \sum_{t=\Delta t}^{k\Delta t} \varepsilon_t + rk\Delta t, \tag{3}$$

where $r = (r_1, r_2, ..., r_n)'$ denotes the one-period foreign money market interest rate. By construction, the uncovered interest rate parity condition is fulfilled for both periods as expected returns across currencies are equalized, namely [9]

$$\mathcal{E}_t(e_{t+\Delta t} - e_t - r\Delta t) = 0. \tag{4}$$

[6] A theoretical foundation would seek to determine the elasticity parameter q_i in a rational expectation model with informed and uninformed currency traders. For the empirical analysis in this paper, a simple reduced form specification is sufficient.

[7] We note that representative agent models appear generally inconsistent with existing evidence for steep asset demand curves as argued by Petajisto (2008). Limited market participation and short-term liquidity supply by financial intermediaries (market makers) may therefore be important market features.

[8] This amounts to a simple scale transformation of the net supply elasticity parameter.

[9] This paper does not focus on uncovered interest rate parity or carry trades motivated by violations of uncovered interest parity.

The exchange rate equilibrium is perturbed by an exogenous demand shock. Such a currency demand shock may come from capital reallocation of index investors following an index revision.

Assumption 2: Currency Demand Shock

At $t = T$, a currency demand shock $u = w^n - w^o$ occurs representing the exogenous capital inflow proportional to the weight changes from old weights $w^o = (w_1^o, w_2^o, ..., w_n^o)'$ to new weights $w^n = (w_1^n, w_2^n, ..., w_n^n)'$.

The demand shock changes the exchange rate. The exact magnitude of the exchange rate effect depends on the short-run supply elasticity captured by the parameters q_i. At time $t = T$, market clearing with $S_i(e_{iT}) = u_i$ implies

$$e_T = \Phi_T + rT + \mathbf{q}^{-1}u, \tag{5}$$

where $\mathbf{q}$ is a diagonal matrix with elements q_i and $\mathbf{q}^{-1}u = (\frac{1}{q_1}u_1, \frac{1}{q_2}u_2, ..., \frac{1}{q_n}u_n)'$. For a small q_i even modest capital flows u_i may generate a large exchange rate effect. An interesting question is to what extent the capital flows imply a permanent currency appreciation. The reduced form model here cannot address this question. Recent evidence on the rebalancing behavior of international equity funds suggest that portfolio managers consider home and foreign equity as imperfect substitutes (Hau and Rey, 2008). This suggests that an index modification should have permanent exchange rate effects.

Assumption 3: Arbitrageurs and Information Structure

A unit interval of currency arbitrageurs with CARA utility and a risk aversion parameter ρ learns about the currency demand shock at the time $t = s < T$. Arbitrageurs undertake optimal arbitrage over the all trading rounds in the time interval $[s, T]$. The exogenous liquidity supply functions $S_i(e_{it})$ are not affected by the demand shock u.

The currency market offers a variety of trading venues and alternatives to the currency spot market. The currency forward or future market may represent preferred instruments of speculative arbitrage (Osler (2008)). Bjonnes and Rime (2005) document that bank dealers often prefer to route their information-based trades through the future market. Contract intermediation by a clearing house provides trade anonymity, which should be important to an informed speculator (Rosenberg and Traub (2008)). Arbitrageurs may therefore implement their trading strategy in the derivative market rather than in the spot market. However, the currency forwards (or futures) and spot markets are highly integrated because 'covered interest parity' generally holds. For the sake of simplicity, the model abstracts from arbitrage opportunities between forward or future and spot markets.

Assumption 4: Infinite Price Elasticity of the Money Market
Interest rate differences $r = (r_1, r_2, ..., r_n)'$ between the foreign and home money market are infinitely price elastic with respect to the speculative trading of FX arbitrageurs.

Assumption 4 is a reasonable approximation if the liquidity supply elasticity in the (short-term) money market exceeds that of the FX market. The speculative demand from the arbitrageurs then leaves the local interest rate unchanged. Forward and future rate returns are then identical to the spot rate returns. The empirical validity of this implication is scrutinized in section 4.2. Next, I turn to the model solution and conjecture testable hypotheses about the price implications of the arbitrage activity.

2.2 Model Solution and Hypotheses

Solving the model is straightforward. The market clearing conditions for all trading rounds take on three different forms given by

$$\begin{aligned} S(e_t) &= 0 \quad \text{for} \quad t < s \\ S(e_t) &= x_t^A \quad \text{for} \quad s \leq t < T \\ S(e_t) &= u \quad \text{for} \quad t = T. \end{aligned} \tag{6}$$

For all trading rounds $t < s$ arbitrageurs are not yet informed about the supply shock and their speculative demand is zero. Arbitrageurs enter the market for the trading rounds $s \leq t < T$ and their optimal demand is denoted by x_t^A. In the last trading round at time $t = T$, the demand shock u occurs and trading stops thereafter.

The CARA utility assumption for the arbitrageurs together with normality of the payoff structure implies linear demand functions. Arbitrage between periods t and $t + \Delta t$ (with $s \leq t < T$) provides a payoff vector characterized by deviations $e_{t+\Delta t} - e_t - r\Delta t$ from the interest parity condition. The risk associated with the arbitrage is given by the covariance matrix $\Sigma\Delta t$ of exchange rate innovations. The optimal demand function of the arbitrageurs under CARA utility then follows as

$$x_t^A = (\rho\Sigma\Delta t)^{-1}\mathcal{E}_t(e_{t+\Delta t} - e_t - r\Delta t). \tag{7}$$

Repeated substitution of the arbitrage demand (7) into the market clearing conditions (6) allows me to solve backward for the equilibrium exchange rate vector e_t until the period $t = s$. For trading round $t < s$ the equilibrium exchange rate follows trivially as $e_t = \Phi_t + rt$. At $t = s$, the exchange rate jumps by $\Delta e_s = e_s - e_{s-\Delta t}$ to the new equilibrium path determined by optimal arbitrage between $t = s$ and $t = T$. Proposition 1 characterizes the movement of the equilibrium exchange rate at $t = s$.

Proposition 1: Spot Exchange Rate Returns

Upon knowledge by the arbitrageurs at time $t = s$ of the index revision from old currency weights w^o to new weights w^n, the spot exchange rate change is positively proportional to the (elasticity-weighted) index change $\mathbf{q}^{-1}(w^n - w^o)$ and negatively proportional to the arbitrage risk term $\Sigma(w^n - w^o)$, where Σ represents the covariance matrix of currency returns. Formally,

$$\Delta e_s \approx \alpha \times \mathbf{q}^{-1}(w^n - w^o) + \beta \times \Sigma(w^n - w^o), \tag{8}$$

with $\alpha = 1 > 0$ and $\beta = -\rho(T - s) < 0$.

Proof: See Appendix.

The term $\mathbf{q}^{-1}(w^n - w^o)$ captures the anticipated price impact of the supply shocks at time $t = T$ in the absence of arbitrage and is referred to as the *fundamental component.* Arbitrage simply moves this component forward in time. The second term $-\rho(T - s)\Sigma(w^n - w^o)$ represents the additional price impact due to their arbitrage risk control. It is referred to as the *risk-hedging component.* The magnitude of the latter depends on the risk aversion parameter ρ and the duration $T - s$ over which the risk is taken on.

Proposition 1 states testable return prediction of the limited arbitrage. Arbitrage risk in the FX market is priced by a negative term $-\rho(T - s)\Sigma(w^n - w^o)$, which requires estimation of the covariance matrix Σ. Only for risk neutrality of the arbitrageurs ($\rho = 0$) should we find $\beta = 0$. Second, the price impact of the weight change $w^n - w^o$ in (8) is scaled by the vector of supply elasticity $\mathbf{q}$, while the hedge term $\Sigma(w^n - w^o)$ is independent of any elasticity parameter. Intuitively, the arbitrageurs chose their optimal hedge so as to equalize the marginal price impact of hedging across all currencies. Illiquid currencies with a very price inelastic supply will attract smaller hedge positions because hedging in these currencies is relatively more expensive. But to correctly capture the price impact of the weight change itself, a proxy for the currency specific supply elasticity q_i is needed.

The model implications for the future rate dynamics are also very simple. Under assumption 4, the vector of foreign interest rates (over the zero home rate) is constant at r. The price of a synthetic (arbitrage free) forward contract for period $t + k\Delta t$ follows simply as

$$f_{t+k\Delta t} = e_t - rk\Delta t. \tag{9}$$

For a constant money market rate difference, the forward market rate $f_{t+k\Delta t}$ should change in step with the spot exchange rate e_t, hence $\Delta f_{s+k\Delta t} = \Delta e_s$ for any k-period forward contract.

Proposition 2: Forward Rate Return

Infinite price elasticity in the money market implies identical event returns for the spot rate and the forward rate. The forward rate change $\Delta f_{s+k\Delta t}$ at the time $t = s$ of the built-up of the arbitrage positions follows as

$$\Delta f_{s+k\Delta t} \approx \alpha \times \mathbf{q}^{-1}(w^n - w^o) + \beta \times \Sigma(w^n - w^o), \tag{10}$$

with $\alpha = 1 > 0$ and $\beta = -\rho(T - s) < 0$.

Proof: Follows directly from assumption 4 and proposition 1.

Proposition 2 implies that the same coefficient estimates should be obtained for the future rate return and for the spot rate return. Alternatively, if the forward rate dynamics substantially deviate from the spot rate dynamics, then either short-term interests are not exogenous or covered interest rate parity is violated. The corresponding evidence is presented in section 4.2.

The propositions 1 and 2 isolate the exchange rate and forward rate dynamics at time $t = s$ when speculators learn about the currency demand shock. Over the consecutive interval $[s, T]$ speculators slowly liquidate their hedging positions and this should reverse the initial exchange rate effect captured by the coefficient β. But the hedge liquidation effect might extend over a longer period and is therefore more difficult to isolate empirically. The empiricial strategy therefore focuses on the exchange rate effects of the speculative position built-up.

3 Data Issues

3.1 The MSCI Index Redefinition

Morgan Stanley Capital International Inc. (MSCI) is a leading provider of equity (international and U.S.), fixed income and hedge fund indices. The MSCI equity indices are designed to be used by a wide variety of global institutional market participants. They are available in local currency and U.S. Dollars (US$), and with or without dividends reinvested.[10] MSCI's global equity indices have become the most widely used international equity benchmarks by institutional investors. By the year 2000, close to 2,000 organizations worldwide were using the MSCI international equity benchmarks. Over US$ 3 trillion of investments were benchmarked against these indices worldwide and approximately US$ 300 to 350 billion were directly indexed. The index with the largest international coverage is the MSCI ACWI (All Country World Index), which includes 50 developed and emerging equity markets, the MSCI World Index (based on 23 developed countries), the MSCI EM (Emerging Markets) Index (based on 27 emerging equity markets), the MSCI EAFE (Europe, Australasia, Far East) Index (based on 21 developed countries outside of North America), the MSCI Europe (based on 14 EU countries (except Luxemburg), plus Norway and Switzerland).

On December 10, 2000, MSCI formally announced that it would adopt a new policy of stock weight calculation based on so-called "free-float" weights. The latter take into account pyramid ownership and control structures in many different countries. They therefore better reflect the limited investibility of many stocks and therefore entire countries. However, the

[10] Aggregating individual securities by different criteria MSCI creates a broad base of indexes such as Global, Regional and Country Equity Indexes, Sector, Industry Group and Industry Indexes, Value and Growth Indexes, Small Cap Equity Indexes, Hedged and GDP-weighted Indexes, Custom Equity Indexes, Real Time Equity Indexes.

formal announcement of the adoption was preceded by an internal decision process and accompanied by a consultation process with the investment community. The first relevant date in this respect dates back to February 2000, when MSCI communicated that it was reviewing its policy on index weights. On September 18, the competing index provider Dow Jones adopted free float weights, thus increasing the pressure on MSCI to take a decision. The next day, MSCI published a consultative paper on possible changes and elicited comments from the investment community. Any adoption decision would be based on the feedback from its clients.

The next important event occurred on December 1, 2000. On that day MSCI announced that it would communicate its decision nine days later on December 10, 2000. The pre-announcement on December 1 presented a strong signal to arbitrageurs that MSCI had taken a decision about the weight change and that public announcement of the index revision was imminent.[11] Strongest arbitrage activity can therefore be expected to occur around this date. Supportive evidence for this interpretation comes from data on the Euro/Dollar spot trading volume in the electronically brokered EBS and Reuters D2000 trading platforms. The first trading day after the pre-announcement (Monday, December 4) is characterized by very large spot trading volume, which exceeds the daily average volume by 30 percent.[12] Some arbitrageurs are likely to have anticipated the free-fload adoption earlier than December 1,

[11]The actual announcement on December 10 seemed to have confirmed the market expectations. Commentators remarked that MSCI's adoption decision was broadly in line with the previous consultation paper. Only the target level of 85 percent of the national market was somewhat higher than expected (by 5 percent) and the implementation timetable was longer than most observers had expected. See again the investment newsletter 'Spotlight on: Throwing Weights Around', Hewitt Investment Group, December 2000.

[12]I examine transaction volumes in the Euro/Dollar spot market available for the period 01/08/2000 to 24/01/2001. Euro/Dollar spot market volume surge should accompany any major international equity reallocation. The data combines all electronically brokered spot contracts in both the EBS and Reuters D-2000 trading platforms on any given day. The first trading day after the pre-announcement, namely Monday, December 4, is characterized by very large spot trading volume of 17,610 contracts. It exceeds the daily average volume by 4,051 contracts or 30 percent. By contrast, trading volume on Monday, December 12, - the first trading day after the second announcement - was below average. The transaction volumes indicate that December 1, 2000 was the relevant news. We thank Paolo Vitale and Francis Breedon for generously providing the transaction data.

and acquired their arbitrage positions prior to this date. Information leakages from MSCI or rational anticipation after the consultation process may have informed arbitrageurs about the likely free float adoption. Interestingly, spot trading volume in the Euro/Dollar rate peaked for example on November 30, 2000, and was 32.5 percent above its quarterly average.

The exact beginning of the arbitrage activity is difficult to date. In order to deal with this issue, alternative event windows are used covering 3, 5 or 7 trading days up to December 4, 2000. The two larger windows start on November 24 and 28, respectively. These longer windows also capture speculative trading before December 1.

3.2 Index Weight Changes and the Arbitrage Risk

The new stock selection criterion based on free float had drastic consequences for the weight of different currencies in the global MSCI index. The absolute weight change $w^n - w^o$ has a standard deviation of 0.006 and appears small. But a more informatived measure is the percentage weight change defined as the weight change $w^n - w^o$ divided by the mean $\frac{1}{2}(w^n - w^o)$ of old and new weights. The percentage weight features a standard deviation 0.326 and is astonishingly large. Figure 1 plots the percentage weight change for each country as a function of the old weights expressed in logs. The Euro Area countries are aggregated to a collective currency weight called Euro Area since these share a common currency.

Figure 1 illustrates that a large number of countries and currencies experienced dramatic reduction in their index representation. For no less that 10 countries the aggregate percentage weight loss exceeded -70 percent (Argentina, Chile, Colombia, Czech Republic, India, Malaysia, Pakistan, Thailand, Turkey, Venezuela) because of a large market share of stock companies with investment restrictions. Most other currencies also experienced index weight losses. The largest absolute weight decreases are registered for Brazil, India, Taiwan, Hong Kong and Mexico. Only 8 countries show a positive country weight change, namely Australia, Egypt, Finland, Greece, Ireland, Morocco, the United Kingdom, and theUnited States. Particularly large and positive are the weight changes for Ireland, the United Kingdom and the United States with percentage weight increases of 11.4 percent, 10.9 percent and 12.0 percent, respectively.

The model of limited arbitrage developed in section 2 implies that arbitrageurs adjust their arbitrage portfolio weights not only to the expected premium proxied by the weight change $w^n - w^o$, but also scale their portfolio weights inversely to the marginal risk $\Sigma(w^n - w^o)$ of each currency position. To estimate the (expected) covariance matrix Σ, I simply use 2 years of daily FX data from July 1, 1998 to July 1, 2000. The exchange rate data is based on end of the day mid-price quotes in London. The estimation period ends 4 months prior to the announcement date of December 1, 2001, so that the estimate of Σ is not affected by the actual event. The 13 countries in the Euro Area share only one common exchange rate after January 1, 1999. Prior to this date I used the ECU currency basket. Argentina and Malaysia feature only very incomplete exchange rate data over the estimation period. Both countries are excluded from the analysis. The total sample includes 37 countries. China and Hong Kong also stand out with currencies of very low exchange rate variation because of their peg to the U.S. dollar. Neither country was excluded from the analysis. However, excluding both countries makes no qualitative difference to the overall results. Table 1, Panel B, provides a summary on the marginal arbitrage risk contribution $[\Sigma(w^n - w^o)]_i$ of the 37 sample currencies. Marginal risk contributions are negative for most currencies. Therefore, increasing the portfolio weights in these currencies provides a hedge against the exposure from excessive dollar investment implied by the strong weight increase for the U.S. currency.

On a more general level, using historical data represents certainly an imperfect measure of the forward looking covariance matrix. But it is also the mostly likely technique used by arbitrageurs to determine the optimal arbitrage strategy and the ex ante risk of their portfolio position. A second more serious concern is the very robustness of the risk estimates based on historical data. Does a weekly sampling frequency alter the estimates of the marginal risk contribution of each currency? To explore this issue the estimation procedure is repeated with weekly spot rate return data. The correlation between estimates based on daily and weekly sampling is 0.984. Figure 2 provides a graphical illustration of this extremely high correlation. Optimal arbitrage portfolios therefore look reasonably similar independently of the sampling frequency of the historic data. Robustness was also checked for a change of the sample period to 3 years or to 18 months. Underlying this relative robustness is the fact that the marginal risk terms only involve the estimation of a $n = 37$ dimensional vector $\Sigma(w^n - w^o)$. Unlike for the estimation of Σ itself, estimating $\Sigma(w^n - w^o)$ does not create a "curse of dimensions". Measurement error with respect to the regressor $\Sigma(w^n - w^o)$ should not to be a serious problem for the analysis.

3.3 Cumulative Event Returns

Before undertaking a more formal regression analysis, it is interesting to examine the time series behavior of exchange rates around the announcement of the MSCI index change. For this purpose the 18 currencies with a percentage weight increase larger than the median are sorted into a group labeled $W+$, while 19 of the most down-weighted currencies are labeled $W-$. On a second sort, the currencies in each group are ranked by their marginal arbitrage risk $\Sigma(w^n - w^o)_i$ into the 9 currencies with the lowest arbitrage risk and therefore highest hedge benefits $H+$ and the remaining currencies with low hedge value labeled $H-$. Four groups of currencies $W+H+$, $W+H-$, $W-H+$, and $W-H-$ are thus obtained. Their respective cumulative average (equal-weighted) return is plotted in Figure 3.[13]

According to the risk arbitrage theory developed in section 2, currencies in group $W+H+$ are the most attractive for speculative long positions and those in group $W-H-$ are most attractive for short positions. Figure 3 shows indeed the predicted cumulative return pattern. The average return for the most desirable currencies in group $W+H+$ increases by more than 3 percent over the 7 trading days from November 24, 2000 to December 4, 2000. Currency returns in group $W-H-$ over the same interval feature a negative average return of 120 basis points. Much of the difference in the cumulative return occurs even before the first announcement on December 1, 2000 and suggests arbitrage trading prior to this date. The strong increase of the cumulative average return for currencies in group $W+H+$ relative to group $W-H-$ from November 24, 2000 to December 4, 2000, validates the event window

[13] To group currencies according to the median weight change provides two equally large subsamples. The focus is on their relative performance. Censoring at the absolute zero weight change is not useful. The arbitrage theory does not state in which currency (U.S. dollar or some currency or currency basket) the arbitrageurs define their objective function. This means that any exchange rate effect can only be predicted up to a fixed effect common to all currencies. Only relative currency effects are of interest here.

selection. This period includes 7 trading days and represents the largest of the 3 event windows considered.

4 Cross Sectional Evidence

The portfolio approach to risk arbitrage developed in section 2 provides clear cross-sectional exchange rate predictions for the arbitrage event. First, the exchange rate appreciation is positively related to the weight change of a country in the portfolio of the global investor. Second, the exchange rate appreciation of a particular currency is negatively affected by its risk contribution to the arbitrage strategy. The following section tests these sign restrictions using a linear panel regressions.

4.1 Evidence on the Spot Rate

The natural correlation of exchange rates suggests a correlated panel approach with one equation for each currency. The linear model is given by

$$\Delta e_{it} = a + b \times D_t + c \times D_t \times \frac{1}{q_i}(w^n - w^o)_i + d \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \qquad E(\mu_t \mu_t') = \Sigma, \quad (11)$$

where the daily (log) exchange rate change Δe_{it} in currency i is regressed on a constant; an event window dummy D_t marking alternatively a 3, 5 or 7 trading day event window around the announcement day of December 1, 2000; the elasticity weighted currency weight change $\frac{1}{q_i}(w^n - w^o)_i$ interacted with the event dummy; and the marginal arbitrage risk contribution $[\Sigma(w^n - w^o)]_i$ of currency i interacted with event dummy. Two different parameterizations are used for the currency supply elasticities q_i. The first specification proxies the currency supply elasticity with the MSCI stock market capitalizations. In particular I chose $q_i = \frac{1}{2}(w^n + w^o)_i$, i.e. the average of the new and old MSCI market weights. The term $\frac{1}{q_i}(w^n - w^o)_i$ then represents the percentage change of a country's index representation. A second elasticity specification is based on FX trading volume in each currency, that is $q_i = Vol_i^{FX}$. The currency specific trading volume is obtained from the BIS trianual market survey of 2001.[14]

[14]Unfortunately, the currency specific trading volume is not available for all currencies in the BIS survey. Where such data is missing I extrapolated the currency trading volume from the FX trading volume

Countries with large equity markets tend to have highly liquid currency markets. This is illustraded by the high correlation of 0.943 between the capitalization based proxy of currency market liquidity and the volume based proxy. Both scaling variables q_i should therefore produce similar results. By contrast, the correlation between the arbitrage risk measure and the scaled weight change is small at 0.29 and 0.21 for the capitalization and volume elasticity proxy, respectively. Regressor colinearity is therefore not a concern.

A general cross currency correlation structure is allowed for the error term μ_{it}. To estimate this error structure more precisely, I use not only the daily data of the event window, but supplement the event window by 2 years of exchange rate data from July 1, 1998 to July 1, 2000. For this period prior to the arbitrage activity the dummy variable D_t takes on the value of zero. All currencies are expressed in dollar terms where $\Delta e_{it} > 0$ denotes the dollar depreciation or foreign currency appreciation. The constant term a captures the average long-run dollar depreciation against all other currencies, while the coefficient b estimates the average dollar depreciation over the event window only. Any particular dollar movement against all other currencies may simply represent a U.S. specific effect and is therefore difficult to interpret. By contrast the coefficient c captures the exchange rate return due to the weight change $(w^n - w^o)_i$ over the event window. Proposition 1 predicts that $c = q^{-1} > 0$. The magnitude of the coefficient c is (inversely) proportional to the elasticity parameter q of the liquidity supply. The term $[\Sigma(w^n - w^o)]_i$ represents the marginal risk contribution of the currency i with its predicted negative effect $d = -\rho < 0$ on the exchange rate return. For the special case when the FX arbitrageurs are risk neutral $(\rho = 0)$, the price effect captured by the coefficient d should be insignificant.

Table 2, presents the regression results for the 37 spot rates in the sample. Regressions in panel A proxy exchange rate elasticities by the average MSCI index representation of a currency, while panel B reports analogous results for currency elasticities proxied by FX trading volume. To evaluate the robustness of the findings, regression results are reported for event windows stretching alternatively over 3, 5 and 7 days. First, I report a baseline regression which excludes the arbitrage risk term. A second specification includes the price

undertaken in the respective country, which is highly correlated with the trading volume of its currency.

effect of the hedging demand. The coefficient a is of no particular interest and not reported in the table.

In the reduced specification in panel A, the fundamental effect $\frac{1}{q_i}(w^n - w^o)_i$ enters statistically significantly at a 1 percent level for all three event windows. But this specification does not control for the risk-hedging demand of the arbitrageurs and may therefore be misspecified. Inclusion of the risk-hedging term reduces the significance level of fundamental term. It remains positive, but only marginally significant in statistical terms. The risk-hedging demand on the other hand has the predicted negative sign at high levels of statistical significance. The adjusted R-squared substantially increases for all windows under inclusion of the risk-hedging term. For example, the five day window in panel A features an adjusted R-squared of 0.43 - an impressive empirical fit for an exchange rate model. Figure 3 illustrates the role of the arbitrage risk visually by plotting the 5 day event return of each currency against the marginal arbitrage risk of each currency. Currencies with very low and negative marginal risk contributions experienced a relative appreciation. A decrease of the arbitrage risk of a currency by one standard deviation or 0.005 implies an average daily currency appreciation of 0.36 ($= 0.005 \times 71.85$) percent or 1.80 percent over the 5 trading days. A comparison of the (relatively imprecise) point estimates c and d suggests that the hedging effect on currencies is large compared to the fundamental effect. The point estimates of d in Panel A are between 65 and 340 times larger than c, while the standard deviation of its regressor is only 65 times smaller. But large standard errors on the coefficients prevent any strong quantitative conclusion.

Panel B reports results for the specification where exchange rate elasticities are proxied by trading volumes. Overall, this alternative specification provides very similar results. The coefficient on the index rebalancing price effect $\frac{1}{q_i}(w^n - w^o)_i$ is positive, but statistically insignificant, while the price impact of the hedging demand $[\Sigma(w^n - w^o)]_i$ has again the correct negative sign and is statistically significant at the conventional 1 percent level. As before, the empirical fit of the model is vastly superior for the full specification. The low level of statistical significance for the fundamental term may be due to either measurement error with respect to the elasticity parameter and/or the small sample size of only 37 observations.

4.2 Evidence on the Forward Rate as a Robustness Check

Speculative positions can be acquired either in the underlying spot market or in the forward or future markets. Speculators may prefer derivative markets to engage in FX arbitrage (Olser, 2008). In this section I verify if the results obtained for currency spot rates extend to the forward market. Forward rates are available from Reuters (via Datastream) as the 4.00 pm U.K. interbank closing rate for the most common maturities of 1 week, 1 month, 3 months and 6 months. The daily forward rate data for these maturities is available for 22 out of 37 currencies. The 22 quoted rates represent the most liquid forward rates.

Before estimating the model implications for forward rates, it is useful to examine the relationship between the different forward rates and the spot rate. Forward rates are generally highly correlated with spot rates and the event period in this study is no exception. The correlation of the spot rate return and the forward rate return over the 7 day event window is above 0.99 for all four forward rate maturities (1 week, 1 month, 3 months and 6 months). The forward market rates and the spot rates react in equal measure to the speculative buying pressures. This also implies that interest rate differences between home and foreign money market rates were not significantly affected by the currency speculation.

The extremely high correlation between the spot rate and forward rate returns make these variables almost interchangeable as event return measures. However, the forward rate sample covers a subset of currencies which are the most liquid ones. They represent the 22 currencies with the highest trading volume. The extremely high liquidity in these currencies may attenuate any speculative buying pressure and render the exchange rate effect of risk arbitrage less pertinent. Repeating the cross-sectional regression for the sample of 22 forward rates therefore presents a useful robustness check for highly liquid currencies.

The linear panel specification remains identical, given by

$$\Delta f_{it} = a + b \times D_t + c \times D_t \times \mathbf{q}^{-1}(w^n - w^o)_i + d \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it} \qquad E(\mu_t \mu_t') = \Sigma. \quad (12)$$

The event period window with its short time series of either 3, 5, or 7 trading days is again complemented by 2 full years of forward rate data to obtain an improved estimate for the cross-sectional correlation structure of exchange rates. The event dummy D_t takes on the value of 1 for the event window and is zero otherwise.

Table 3 reports the regression results for the 1 week forward rates for the two elasticity specifications. Panel A proxies elasticities by the MSCI market weights and Panel B by FX trading volumes. As in Table 2, the fundamental effect $\frac{1}{q_i}(w^n - w^o)_i$ has a positive sign and the risk-hedging term $[\Sigma(w^n - w^o)]_i$ the expected negative sign. The term $\frac{1}{q_i}(w^n - w^o)_i$ is now statistically significant at the 5 percent level for both the 3 and 7 day windows. The adjusted R-squared for the smaller sample of 22 forward rates is still higher than for the larger sample of 37 spot rates. For the 3 day window I find an adjusted R-squared of more than 57 percent for the announcement event returns. The same regression is repeated with 1 week and 3 month forward rates and the results are very similar (but not reported). I conclude that currency hedging components of speculative trading can be found both in the spot rate returns and the forward rate returns. Speculative risk hedging demands are price relevant for the entire sample of all currencies, but also for the subsample of the most liquid currencies.

5 Spectral Implications of Multi-Currency Arbitrage

While the statistical evidence in the previous section is supportive of the model and provides a very good empirical fit to the data, it falls short of providing high statistical significance levels for the model parameter values. This is not surprising given the small sample size for which the model is tested. Exchange rates are notoriously volatile and even a relatively short event window incorporates many other confounding exchange rate effects which cannot be controlled for and hence enter the error terms of the regression. The following section develops a methodology which extracts the speculators' trading pattern at the microstructure level by focusing on the high frequency return cospectrum of currency pairs. The spectral analysis provides a magnifying glass which allows me to focus on the joint high frequency dynamics of the exchange rates and thus identify speculative trading more distinctly.

5.1 Methodology and Data

The additional spectral implications for the exchange rate dynamics derived in this section are closely related to the implementation of multi-asset risk arbitrage strategies. If risk management is an important element of multi-asset risk arbitrage, then *simultaneous* implementation of the speculative positions across all currencies should be common practice. Risk arbitrageurs should build up the optimal arbitrage positions in parallel across all currencies. But this implies that any price impact of their trading strategy is also simultaneous. We can therefore predict a particular cross-sectional pattern of high-frequency return comovements which corresponds to the vector of optimal arbitrage positions. Comovement in statistics is generally captured by the covariance and can be decomposed into its different frequency components called cospectrum. Formally, the sample covariance between two demeaned time series X_s and Y_s has the frequency domain representation

$$Cov(X_s, Y_s) = \frac{1}{S}\sum_{s=1}^{S} X_s Y_s = \frac{1}{2}\sum_{f=1}^{N}\left(\alpha_f^X \alpha_f^Y + \beta_f^X \beta_f^Y\right) = \sum_{f=1}^{N} Cosp_{XY}(f), \tag{13}$$

where (for an odd number S) each of the $N = (S-1)/2$ cospectrum terms $Cosp_{XY}(f)$ represents the contribution of comovements at frequency f to the total comovement or covariance.[15] The cospectrum terms follow directly from a discrete Fourier transformation of the individual series X_s and Y_s, where α_f^X and α_f^Y are coefficients of the cosine components and β_f^X and β_f^Y are the coefficients for the sinus components. The additivity of the cospectrum allows us to define spectral bands which aggregate certain frequencies in a frequency band. For the purpose of the analysis, four different spectral bands $B = \{VH, H, M, L\}$ are defined, which decompose the covariance of any pair (i, j) of exchange rate returns Δe_{is} and Δe_{js} into its very high, high, medium and low frequency cospectrum; hence

$$Cov(\Delta e_{is}, \Delta e_{js}) = \sum_{f=1}^{N} Cosp(i, j, f) = \sum_{B=VH,H,M,L} Cosp(i, j, B). \tag{14}$$

The double sorting of currencies from section 3.3 into (relatively) up- and downweighted $(W+/W-)$ and those with positive or negative hedge value $(W+/W-)$ is useful gain.

[15] For a details, see Hamilton (1994), page 275.

Currency pairs (i, j) drawn from the most desirable arbitrage currencies $(W + H+)$ should be subject to simultaneous joint buying by the speculators, which should generate an increase in the high frequency comovement measured by the very high frequency cospectrum band $Cosp(i, j, VH)$. Simultaneous selling of two currencies from the group of least desirable arbitrage currencies $(W - H-)$ should also generate a positive high frequency return comovement as both currencies are expected to show a negative return due to joint short selling. However, (cross group) currency pairs where one currency is drawn from the groups $(W+H+)$ and the other from the group $(W-H-)$ should show a more negative comovement for the event period. In order to compare the cospectrum of the event period to the ordinary cospectrum at regular times, I use a control period of the same length as the event period. The change in the cospectrum or spectral shift of the frequency band B is then defined as

$$\Delta Cosp(i, j, B) = Cosp(i, j, B)^{Event} - Cosp(i, j, B)^{Control}, \tag{15}$$

where $Cosp(i, j, B)^{Event}$ and $Cosp(i, j, B)^{Control}$ denotes the cospectrum of the event and control period, repetitively. The remainder of the analysis focuses on changes in the cospectrum relative to the natural cospectrum for any currency pair. Currency pairs with arbitrage positions in the same direction (both buy or both sell) should be characterized by positive spectral shift in the high frequency cospectrum, while currency pairs with arbitrage positions in opposite directions (one buy, the other sell) should feature a negative shift in the high frequency cospectrum.

Proposition 3: Simultaneous Implementation of Arbitrage Positions
Simultaneous implementation of speculative positions by a sequence of risk arbitrageurs implies a specific modification of the high frequency components of the cospectrum for each currency pair (i, j). For the event period, the shift in the cospectrum $\Delta Cosp(i, j, B)$ is (i) detectable in the highest spectral band $B = VH$ and (ii) proportional to the product $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of event period exchange rate returns.
Proof: See Appendix.

Testing proposition 3 requires high frequency data for a large cross section of exchange rates. Such data is obtained from the commercial data provider Olsen Associates for the three months from September to December 2000. Olsen systematically records FX bid and ask quotes from the Reuters terminal at one minute intervals. Such high frequency data was available for all of the 37 currencies except China and Sri Lanka. The spectral analysis is based on midprices calculated as the arithmetic average of the bid and ask price at the end of each minute interval. If no bid or ask price is available for a one minute interval, the last available quote is used to calculate the midprice. The event period is given by the same 7 trading days from November 24 to December 4 for which the cross-sectional evidence was presented. As control period I use the period from September 8 to September 18, 2000, which also covers 7 trading days.

Both the event or arbitrage period and the control period comprise a total of $10,080$ $(= 60 \times 24 \times 7)$ one minute intervals. A large number of intervals do not feature new price quotes. In this case I assume that the previously quoted bid and ask are still valid and the midprice is therefore unchanged. Highly liquid markets like the Euro-Dollar rate or the Yen-Dollar rate have new quote arrivals for approximately 90 percent of the one minute intervals. On the other end of the scale we find the Egyptian Pound with new quotes in its dollar rate in only 1 percent of the $10,080$ intervals. A second feature of the high frequency exchange rate returns is their high negative autocorrelation for lags up to 5 minutes. Pooling all exchange rates produces a partial autocorrelation at lag 1 of -0.258 (-0.237) for the event (control) window and -0.105 (-0.088) at lag 2. The strong negative serial correlation of the midprice indicates short-run overshooting behavior of the exchange rate. Intuitively, demand shocks remove some of the liquidity supply on one side of market and it may take time for new best indicative quotes to replace the absorbed liquidity. Importantly, such strong negative serial correlation due to trading events will tend to leave a particularly large footprint in the high frequency spectrum of the exchange rate return process.[16] It also implies that the cospectrum of currency pairs is particularly pronounced at the highest frequencies if both currencies experience simultaneous trading action. The speculative multi-asset trading strategies should therefore be most detectable at very high frequencies and this motivates the spectral analysis.

[16] *Positive* serial correlation of an MA(1) or AR(1) process for example implies a spectral density function which is decreasing in the frequency spectrum, while *negative* serial correlation implies a spectral density function which is increasing for higher frequencies.

As the "very high" frequency band I define the sum of the 15 highest frequencies, which capture return simultaneity within a 15 minute interval. The next 45 highest frequencies are aggregated into the "high" frequency cospectrum, thus representing comovements between 15 minutes and 1 hour. "Medium" frequency comovements combine the frequencies from 1 hour to 4 hours and all lower frequencies are aggregated into the "low" frequency cospectrum. The exact segmentation of the frequency band is somewhat arbitrary. However, the results reported in the following section are robust to alternative (though qualitatively similar) segmentations of the frequency spectrum.

5.2 Spectral Evidence for Currency Groups

First, I explore the spectral evidence for the most attractive "buy" currencies combined in the portfolio $W+H+$ and the least attractive "sell" currencies in the portfolio $W-H-$. For currency pairs (i,j) where both currencies are drawn from the same portfolio, I expect a positive change in the high frequency cospectrum due to the simultaneous buying or selling. By contrast, a negative comovement in the high frequency spectrum is expected if currency i is drawn from portfolio $W+H+$ and currency j from portfolio $W-H-$. Cross-group currency pairs with buying activity in currency i and selling in currency j imply a negative shift in the high frequency spectrum. Table 4 reports the evidence on the cospectrum for currency pairs formed within groups $W+H+$ and $W-H-$ in panel A and across groups in panel B. The cospectrum is stated separately for the event period, the control period and the spectral difference between both. The Wilcoxon sign-ranked test reports if the difference in the cospectrum is statistically significant for any of the 4 spectral frequency bands. In panel A, a positive cospectrum is found for both event and control period at all 4 frequency bands. Interestingly, the spectral change is concentrated in the very high frequency bands and features the expected positive sign. The Wilcoxon sign-rank test strongly rejects the hypothesis that the median is the same for the event and control sample. Currency pairs for which joint buying or joint selling is the optimal arbitrage strategy clearly show a stronger high frequency comovement over the event period. For the remaining three spectral bands the change in the cospectrum is statistically insignificant. In panel B, the high frequency spectrum also features the largest change between event and control period. The very high frequency spectral shift is negative as expected since the currency pairs in panel B combine exchange rates for which the optimal arbitrage portfolio prescribes long and short positions. Simultaneous buying and selling should therefore generate a negative covariance and a negative spectral shift. Panel C reports additional sign tests for the relationship between pair type (within group or across group) and the sign of the spectral shift. For the very high frequency band, the 68 within group currency pairs show an increased cospectrum in 45 cases, while the 72 cross group pairs show an increased cospectrum for only 27 cases. The Fisher test indicates a clear statistical association between pair type and the sign count. No such sign correlation is detectable for the lower frequency bands. The particular return patter of arbitrage trading becomes most visible in the very high frequency domain. A graphical illustration of the evidence in Table 4 is presented in Figure 5. The change in the covariance within and across groups is concentrated in the very high frequency band which measures comovements within a 15 minute interval. Overall, the evidence shows that the covariance change across the sorted currency pairs has the predicted sign and is statistically highly significant if the analysis focuses on the very high frequency cospectrum.

5.3 Spectral Band Regressions

Next, I explore whether the high frequency spectral shift for each currency pair (i, j) is indeed proportional to the corresponding product $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of expected exchange rate changes as expressed in proposition 3. The expected exchange rate change in currencies i and j is linear in the two parameters c and d according to

$$\Delta\widehat{e}_i(c,d) = c \times q_i^{-1}(w^n - w^o)_i + d \times [\Sigma(w^n - w^o)]_i \quad (16)$$

$$\Delta\widehat{e}_j(c,d) = c \times q_j^{-1}(w^n - w^o)_j + d \times [\Sigma(w^n - w^o)]_j, \quad (17)$$

where $q_i^{-1}(w^n - w^o)_i$ represents again the fundamental effect and $[\Sigma(w^n - w^o)]_i$ the risk-hedging effect. The change in the cospectrum $\Delta\overline{Cosp}(i, j, B)$ is explained by the quadratic form $\Delta\widehat{e}_i(c, d) \times \Delta\widehat{e}_j(c, d)$, hence

$$\Delta Cosp(i, j, B) = \Delta\widehat{e}_i(c, d) \times \Delta\widehat{e}_j(c, d) + \epsilon_{ijB} \quad \text{for} \quad B = VH, H, M, L. \quad (18)$$

This quadratic model is estimated using a maximum likelihood method. As for the event return, the fundamental effect has a positive coefficient, hence $c > 0$, and the risk-hedging effect a negative coefficient, thus $d < 0$. But the left hand side variable is now given by the cospectrum, which increases the number of observations to all currency pairs and furthermore allows a separate regression for each spectral band. Under simultaneous implementation of the arbitrage strategy, the best regression fit is expected for the highest frequency band. The very high frequency band also aggregates the smallest number of Fourier coefficients. Only the sine and cosine coefficients of the 15 highest frequency are used, which implies a total of 1050 ($= 2 \times 15 \times 35$) Fourier coefficients for 35 currencies. The 1050 Fourier coefficients fully characterize the high frequency behavior of all 35 exchange rate return series. I note that aggregation of these Fourier coefficients into 595 different cospectrum pairs still implies strictly less degrees of freedom for the dependent variable than the raw data features.[17]

Table 5 reports the spectral band regressions for each of the 4 spectral bands. Panels A and B use the elasticity specification based on equity market weights, while panels C and D proxy the exchange rate elasticity by the FX trading volume. For both specifications, different samples of exchange rate pairs are used. Panels A and C use the full sample of all $n = 595$ currency pairs, while panels B and D estimate the regression for the $n = 231$ currency pairs consisting of the 22 most liquid currencies. For all 4 panels, a qualitatively similar result is obtained. The regression for the very high frequency band in each panel features statistically highly significant coefficient estimates with the predicted, namely a positive coefficient c for the fundamental effect and negative point estimate d for the hedging effect. No significant relationship is found for the three other spectral bands.

For the very high frequency band the regression fit is very good, and particularly so for the subset of very liquid currency pairs in panels B and D. In panel B, the two independent variables explain 48 percent of the high frequency spectral shift of the 231 currency pairs. Figure 6 provides a graphical illustration of Panel B in which the very high frequency cospectrum shift $\Delta Cosp(i, j, VH)$ is plotted against the product $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$ of predicted exchange rate changes. The t-values for the corresponding coefficient estimates c and d are

[17] In other words, the higher statistical signifcance obtained for the spectral band regression is not an artefact of replicating data observations through the formation of currency pairs.

7.02 and −28.27, respectively. The spectral evidence with its focus on the high frequency comovements therefore allows us to assert the existence of the fundamental and risk-hedging component at much higher levels of statistical significance than the conventional evidence in Table 3.

It is also instructive to compare the points estimates c and d as quantitative measures of the fundamental effect and the hedging effect, respectively. Arbitrage risk as a regressor features a 65 times smaller standard deviation compared to the percentage weight change. However, the point estimate for d is (in absolute value) roughly 200 times larger than c in the very high frequency band. The ratio of point estimates here is surprisingly close to the results in Table 3, Panel A, which show d to be 170 times large than c for the same 7 day data period and the same subsample of highly liquid currencies. But considerably smaller standard errors on the coefficients in the spectral band regressions allows a quantitative assessment: The exchange rate effect of hedging demands in the MSCI arbitrage episode are at least as large (or even larger) than the arbitrage related fundamental adjustment.

I conclude that the spectral analysis represents a very useful statistical tool to identify the structure and price impact of multi-asset trading strategies. Simultaneous trading across many assets by different investors leaves a unique 'footprint' in the cospectrum.

6 Conclusion

Global capital flows may create speculative opportunities in currency markets whenever such flows are large and predictable. Examples include asset liquidations of large international investors, capital market deregulation which opens markets to a new set of investors, changes in international interest differentials or a global index revision such as the one considered in this paper. The analysis of a shock to multiple currencies require a portfolio approach. Risk averse optimizing arbitrageurs take speculative positions which optimize a trade-off between expected return and arbitrage risk. Optimal arbitrage positions depend positively on the expected arbitrage premium, but negatively on the marginal risk contribution of any arbitrage position to overall arbitrage risk. Intuitively, a currency with a negative correlation to all other appreciating currency can be arbitraged at a much lower risk than another appreciation currency for which the same correlations are mostly positive. For risk averse speculators, the optimal FX investment strategy is jointly determined by the expected return premium resulting from the capital reallocation and the correlation structure of all exchange rates. The price effect of arbitrage can be decomposed into a fundamental component and a risk-hedging component. Evidence for the existence of the risk-hedging component is direct evidence of the risk aversion of the arbitrageurs.

The empirical part of the paper uses a unique natural experiment to test these predictions. The redefinition of the MSCI global equity index in 2001 and 2002 provides an exogenous shock to global equity allocations. The index revisions induced enormous changes in the index representation of various countries and consecutively implied large international capital flows by index tracking equity funds. First, a conventional event study approach is used in which daily returns over different event windows are regressed on the predicted fundamental component and on the risk-hedging component. The fundamental effect is marginally significant with the predicted positive sign, while the evidence for the risk-hedging effect is much stronger. The evidence is confirmed for the subsample of forward rates which feature virtually identical event returns for the most common maturities. The point estimates indicate an economically significant currency effect for the transitory hedging demand of 3.6 percent return difference for a two standard deviation change in the hedging benefit of a currency. But a clear limitation of the evidence is that the sample size of only 37 currencies implies large standard errors for the point estimates.

The second part of the paper develops a new approach which uses high frequency data combined with spectral analysis to obtain stronger statistical results. The idea is straightforward. Simultaneous implementation of a multi-currency arbitrage position by a random sequence of investors implies a distinct shift in the high frequency cospectrum across currency pairs. The spectral shift of the high frequency cospectrum for all currency pairs should be proportional to the product of the predicted exchange rate changes. The spectral band regressions allow for a much more precise inference since "low frequency noise" is filtered. The spectral approach provides very strong statistical evidence in favor of the model of limited arbitrage. Both the fundamental component and the risk-hedging component are statistically highly significant in explaining the cospectrum shift for the 15 highest Fourier frequencies, showing that arbitrage trading occurred simultaneously across currencies. The spectral inference also allows a more precise qualitative assessment of the role of hedging demands during speculative episodes. The MSCI event window reveals that speculative position hedging had a (short-term) exchange rate effect which was at least as large (or possibly larger) than the fundamental effect towards the new equilibrium. This important role of hedging demands in the MSCI index event is direct evidence that currency arbitrage capital is rather limited (and risk averse). Such a finding supports the hypothesis that the empirical failure of uncovered interest rate parity is also due to insufficient arbitrage capital.

Overall, risk arbitrage in the currency market appears correctly captured by the portfolio approach. Risk aversion of the investors explains why the hedging or arbitrage risk component is a very significant pricing factor over the arbitrage period. On a mythological level, I highlight that the increasing availability of high frequency data allows for better statistical inference about multi-asset arbitrage strategies. The segmentation of the frequency domain can substitute for cross-sectional sample size and allow for sharper inference as exemplified in this paper.

Appendix

Proposition 1:

Let e_t denote a vector of currencies $i = 1, 2, 3, ..., n$. The time interval $[0, T]$ is partitioned into N equal intervals $\Delta t = T/N$ and trading occurs at times $t = 0, \Delta t, 2\Delta t, 3\Delta t, ..., T$. Speculators learn about the supply shock $u = w^n - w^o$ at time $t = s < T$. The market clearing condition requires

$$\begin{array}{rcll} \mathbf{q}(e_t - \Phi_t - rt) & = & 0 & \text{for} \quad t < s \\ \mathbf{q}(e_t - \Phi_t - rt) & = & (\rho\Sigma\Delta t)^{-1}\mathcal{E}_1(e_{t+\Delta t} - e_t - r\Delta t) & \text{for} \quad s \leq t < T \\ \mathbf{q}(e_t - \Phi_t - rt & = & u & \text{for} \quad t = T, \end{array} \tag{19}$$

where $\mathbf{q}$ represents the diagonal matrix $(n \times n)$ with the currency specific supply elasticities as elements. Taking differences between the equilibrium conditions for $t = T$ and $t = T - \Delta t$ and applying expectation operator $\mathcal{E}_{T-\Delta t}$ on both sides gives

$$\mathbf{q}\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} - r\Delta t) = u - (\rho\Sigma\Delta t)^{-1}\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} - r\Delta t) \tag{20}$$

and solving for the expected exchange rate return yields

$$\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} - r\Delta t) = \left[\mathbf{I} + (\mathbf{q}\rho\Sigma\Delta t)^{-1}\right]^{-1} \mathbf{q}^{-1}u. \tag{21}$$

Substitution of (21) into the market clearing condition (19) at time $t = T - \Delta t$ implies

$$\begin{array}{rcl} e_{T-\Delta t} & = & \Phi_{T-\Delta t} + r(T - \Delta t) + (\mathbf{q}\rho\Sigma\Delta t)^{-1}\,\mathcal{E}_{T-\Delta t}(e_T - e_{T-\Delta t} - r\Delta t) \\ & = & \Phi_{T-\Delta t} + r(T - \Delta t) + [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1}\,\mathbf{q}^{-1}u. \end{array} \tag{22}$$

For a small Δt we can use the linear approximation $[\mathbf{I} - \mathbf{q}\rho\Sigma\Delta t] \approx [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1}$and simplify

$$e_{T-\Delta t} \approx \Phi_{T-\Delta t} + r(T - \Delta t) + \mathbf{q}^{-1}u - \rho\Delta t(\Sigma u). \tag{23}$$

The equilibrium condition for the periods t with $s \leq t < T - \Delta t$ follows by repeated substitution. Starting with the market clearing condition (19) for $t = T - 2\Delta t$,

$$\mathbf{q}(e_{T-2\Delta t} - \Phi_{T-2\Delta t} - r(T - 2\Delta t)) = (\rho\Sigma\Delta t)^{-1}\mathcal{E}_{T-2\Delta t}(e_{T-\Delta t} - e_{T-2\Delta t} - r\Delta t), \tag{24}$$

substitution for $e_{T-\Delta t}$ yields

$$e_{T-2\Delta t} - \Phi_{T-2\Delta t} - r(T-2\Delta t) = [\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1} (\mathbf{q}^{-1}u - \rho\Delta t(\Sigma u). \tag{25}$$

Using the approximation $[\mathbf{I} + \mathbf{q}\rho\Sigma\Delta t]^{-1} \approx [\mathbf{I} - \mathbf{q}\rho\Sigma\Delta t]$ again and ignoring terms of order $(\Delta t)^2$ implies

$$e_{T-2\Delta t} \approx \Phi_{T-2\Delta t} + r(T-2\Delta t) + \mathbf{q}^{-1}u - \rho 2\Delta t(\Sigma u) \tag{26}$$

Repeated backward substitution for all t up to $t = s$ yields

$$e_s \approx \Phi_s + rs + \mathbf{q}^{-1}u - \rho(T-s)(\Sigma u). \tag{27}$$

The exchange rate change at time $t = s$ follows as

$$\Delta e_s = e_s - e_{s-\Delta t} \approx \mathbf{q}^{-1}u - \rho(T-s)(\Sigma u). \tag{28}$$

An exact solution can be determined in the limit case with $\Delta t \to 0$. This amounts to solving the system of first-order stochastic differential equations characterized by

$$de_t = rdt + \rho\mathbf{q}\Sigma(e_t - \Phi_t - rt)dt + d\varepsilon_t, \tag{29}$$

with $\Phi_t = \int_{s=0}^{t} d\varepsilon_t$. Instead of a term $\rho(\Sigma u)(T-t)$ linear in t, the dynamic adjustment towards T is governed by a linear combination $\sum_{i=1}^{n} A_i e^{\lambda_i t}$, where the coefficients λ_i denote the eigenvalues of the matrix $\rho\mathbf{q}\Sigma$. The two boundary conditions

$$e_T = \Phi_T + rT + \mathbf{q}^{-1}u \tag{30}$$

$$\mathcal{E}_t \left.\frac{de_t}{dt}\right|_{t=T} = r + \rho(\Sigma u) \tag{31}$$

hold. The solution in equation (27) represents a linear approximation to the exact limit case with $\Delta t \to 0$. At $t = T$ the two solutions coincide in levels and in the first time derivative. This means that the linear approximation is good as long as the risk arbitrage period $T - s$ is short and the eigenvalues λ_i are small. The eigenvalues are small if the risk aversion ρ is small.

Proposition 3:

So far it was assumed that all arbitrageurs learn about the supply shock u simultaneously and acquire an arbitrage position instantaneously at time $t = s$. Consider now the case in which arbitrageurs built their arbitrage positions sequentially over trading rounds $s = 1, 2, ...S$ in the event window. Assume $r = 0$ for simplicity. Let Δe_s^{Event} denote the exchange rate return process over the event period of S intervals and Δe_s^{Contol} the exchange rate return process for an equally long control period. In accordance with the model it is assumed that the exchange rate effect of a persistent speculative demand shock is linear in size and also persistent. In the absence of arbitrage trading, the n currency prices (in logs) follow a random walk $\Delta e_s = e_s - e_{s-1} = \varepsilon_s$ with $\mathcal{E}(\varepsilon_s) = 0$ and $\mathcal{E}_{s-1}(\varepsilon_s \varepsilon_s') = \Sigma$. The return covariance between a currency pair (i, j) is denoted by Σ_{ij}. Under the null hypothesis of no speculative activity in the event period, the covariance change between the event and the control period is zero. Formally,

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) - cov(\Delta e_{is}^{Control}, \Delta e_{js}^{Control}) = \Sigma_{ij} - \Sigma_{ij} = 0. \tag{32}$$

Similarly, if the stochastic process Δe_s is the same over the event and control period, then the difference of the respective cospectra $\Delta Cosp(i, j, f)$ should be zero for any currency pair and all frequencies f, i.e.

$$\Delta Cosp(i, j, f) = Cosp(i, j, f)^{Event} - Cosp(i, j, f)^{Control} = 0. \tag{33}$$

Consider next the case of speculative activity in the event period. A sequence of $k = 1, 2, .., K < S$ arbitrageurs trade (each once) sequentially in trading rounds $s(1), s(2), ..., s(K) \leq S$. Each arbitrageur has a relative size θ_k so that his price impact is given by

$$\theta_k \Delta \widehat{e}_{s(k)} = \theta_k \left\{ c \times \frac{1}{q_i} (w^n - w^o)_i + d \times [\Sigma(w^n - w^o)]_i \right\}. \tag{34}$$

The combined size of all traders is scaled to one, hence $\sum_{k=1}^{K} \theta_k = 1$. The covariance for the event period under speculative activity follows as

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) = \Sigma_{ij} + \frac{1}{S} \sum_{k=1}^{K} \theta_k^2 \left[\Delta \widehat{e}_{is(k)} \times \Delta \widehat{e}_{js(k)} \right]. \tag{35}$$

The covariance change between the event and control period therefore follows as

$$cov(\Delta e_{is}^{Event}, \Delta e_{js}^{Event}) - cov(\Delta e_{is}^{Control}, \Delta e_{js}^{Control}) = \frac{\vartheta(0)}{S}\overline{\theta^2}\,[\Delta\widehat{e}_i \times \Delta\widehat{e}_j] = \gamma_{ij}(0), \qquad (36)$$

where an expected impact parameter is defined as $\overline{\theta^2} = \frac{1}{K}\sum_{k=1}^{K}\theta_k^2$ and the number arbitrage trading events as $\vartheta(0) = K$. Let $\gamma_{ij}(h)$ denote the covariance shift corresponding to a lag of h trading rounds. The change in the cospectrum at a particular frequency f follows as

$$\Delta Cosp(i,j,f) = \frac{2}{S}\sum_{h=-S+1}^{S-1}\gamma_{ij}(h)\cos(h\omega_f). \qquad (37)$$

Assume that each arbitrageur k trades only once and his trading period $s(k)$ represents an independent drawn from a uniform distribution over all S trading opportunities. The likelihood of two arbitrageurs trading at an interval of h periods ($0 < |h| \leq S-1$) is given by $\frac{2}{S^2}(S-h)$. Moreover, for K arbitrageurs there are $K(K-1)/2$ pairs of arbitrageurs which could trade at interval h. The expected number of trading events for the two series Δe_{is} and Δe_{js} at lag $h \neq 0$ among K arbitrageurs follows as

$$\vartheta(h) = \frac{K(K-1)(S-|h|)}{S^2}. \qquad (38)$$

A parameter defined as

$$\underline{\theta^2} = \frac{1}{K(K-1)}\sum_{k=1}^{K}\sum_{l=1,l\neq k}^{K}\theta_k\theta_l, \qquad (39)$$

characterizes the joint expected covariance impact of two different arbitrageurs ($l \neq k$). The event period cross covariance between Δe_{is} and Δe_{is-h} lagged by $h \neq 0$ trading rounds follows as

$$\gamma_{ij}(h) = cov(\Delta e_{is}^{Event}, \Delta e_{js-h}^{Event}) = \frac{\vartheta(h)}{S}\underline{\theta^2}\,[\Delta\widehat{e}_i \times \Delta\widehat{e}_j] \qquad (40)$$

The cospectrum (for $\omega_s = 2\pi f/S$) is characterized as

$$\begin{aligned}\Delta Cosp(i,j,f) &= \frac{2}{S}\sum_{h=-S+1}^{S-1}\gamma_{ij}(h)\cos(h\omega_f) = \frac{2}{S}\gamma_{ij}(0) + \frac{4}{S}\sum_{h=1}^{S-1}\gamma_{ij}(h)\cos(h\omega_f) \\ &= \left\{\frac{2}{S}\frac{K}{S}\overline{\theta^2} + \frac{4}{S}\sum_{h=1}^{S-1}\frac{\underline{\theta^2}K(K-1)(S-h)}{S^2}\cos(h\omega_f)\right\}[\Delta\widehat{e}_i \times \Delta\widehat{e}_j] \\ &\approx \frac{2}{S}\frac{K}{S}\overline{\theta^2}\,[\Delta\widehat{e}_i \times \Delta\widehat{e}_j] \qquad (41)\end{aligned}$$

based on the approximation

$$\sum_{h=1}^{S-1} \frac{(S-h)}{S} \cos(h\omega_f) \approx \int_0^1 (1-x)\cos(2\pi f x)dx = 0. \tag{42}$$

The cospectrum change $\Delta Cosp(i,j,f)$ is constant in f and proportional to $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$ for every frequency f. This implies that the 'very high' frequency cospectrum shift $\Delta Cosp(i,j,B = VH)$ is also proportional to $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$. The result is obtained under the assumption that speculative demand shocks generate a linear exchange rate effect without serial return correlation. Empirically, however, currency trading generates a negative serial correlation for currency returns. The midprice between the best ask and bid quotes tends to briefly overshoots. Under negative serial correlation, high frequency components of the cospectrum capture a relatively larger proportion of the overall covariance in any currency pair. Hence, the speculative trading pattern is most pronounced in the highest spectral band $B = VH$.

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Table 1: Summary Statistics on Panel Data

Reported are summary statistics for the dependent and independent variables in Panels A and B, respectively. Panel data on daily exchange rate returns ($n = 37$) and forward rate returns returns for the 1 month forward rate ($n = 21$) are reported for the 7 day event window complemented by 2 years of return data from July 1, 1998 to July 1, 2000. The cospectrum shift is calcluated for the 7 day event window relative to a 7 day control window using minute by minute return data from Olsen Associates for $n = 35$ currencies.

			Obs.	Mean	S.D.	Min	Max
Panel A: Dependent Variables							
Exchange rate returns		Δe_{it}	15,096	−0.035	0.584	−11.770	5.993
Forward rate returns		Δf_{it}	8,477	−0.029	0.612	−5.156	5.137
Cospectrum shift							
Very high frequency band	$(\times 10^9)$	$\Delta Cosp(i,j,VH)$	595	0.865	17.60	−92.80	59.83
High frequency band	$(\times 10^9)$	$\Delta Cosp(i,j,H)$	595	−1.382	14.43	−67.46	52.07
Medium frequency band	$(\times 10^9)$	$\Delta Cosp(i,j,M)$	595	−0.226	11.30	−55.86	51.02
Low frequency band	$(\times 10^9)$	$\Delta Cosp(i,j,L)$	595	−1.823	8.35	−74.14	27.94
Panel B: Independent Variables							
Weight change		$(w^n - w^o)_i$	37	−0.0016	0.0060	−0.0328	0.0107
Elast. 1 × weight change		$\frac{2}{(w^n+w^o)_i} \times (w^n - w^o)_i$	37	−0.4314	0.3259	−0.9825	0.1086
Elast. 2 × weight change	$(\times 10^6)$	$\frac{1}{Vol_i^{FX}} \times (w^n - w^o)_i$	37	−0.3028	0.4009	−1.6812	0.0688
Marginal risk contribution		$[\Sigma(w^n - w^o)]_i$	37	−0.0041	0.0050	−0.0143	0.0010

Table 2: Panel Regressions for Daily Spot Exchange Rate Returns

The (log) daily spot exchange rate returns Δe_{it} (denominated in dollars per local currency and expressed in percentage points) is regressed on a constant, a time dummy D_t marking the event window, the time dummy interacted with the product of the supply elasticity q^{-1} and MSCI weight change $(w^n - w^o)_i$ of all stocks in currency i and the time dummy interacted with the risk contribution $[\Sigma(w^n - w^o)]_i$ of a currency to the arbitrage portfolio. Formally,

$$\Delta e_{it} = a + b \times D_t + c \times D_t \times q_i^{-1}(w^n - w^o)_i + d \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \qquad E(\mu_t \mu_t') = \Omega.$$

The time period covers 2 years of daily exchange rate return in 37 currencies for the period of July 1, 1998 to July 1, 2000 and the additional event window. Reported are results for event windows of 3, 5 and 7 trading days. Panel A reports results where the currency specific elasticity q_i is proxied by the relative market capitalization, i.e. the average old and new index weights $q_i = \frac{1}{2}(w^n + w^o)_i$. Panel B proxies the the same regressions where the elasticity is proxied by the currency specific trading volume according to the BIS 2001 trianual market survey, i.e. $q_i = Vol_i^{FX}$. The constant coefficient estimate a is not reported. Panel corrected z-values are reported in parenthesis. The adjusted R^2 states the explanatory power for the event window period.

Event Window	b	[z]	c	[z]	d	[z]	Adj. R^2
Panel A: Capitalization Based Exchange Rate Elasticities (Spot Rates Returns, N=37)							
3 Days	0.63	[4.43]	0.57	[3.54]			0.330
3 Days	0.19	[0.12]	0.24	[1.71]	−81.44	[−2.91]	0.545
5 Days	0.44	[3.93]	0.41	[3.30]			0.206
5 Days	0.05	[0.53]	1.12	[1.12]	−71.85	[−3.32]	0.433
7 Days	0.42	[4.46]	0.43	[4.04]			0.193
7 Days	0.10	[1.37]	0.20	[2.08]	−58.78	[−3.21]	0.354
Panel B: Volume Based Exchange Rate Elasticities (Spot Rate Returns, N=37)							
3 Days	0.49	[3.87]	0.32	[3.20]			0.299
3 Days	0.12	[1.04]	0.13	[1.39]	−84.15	[−3.02]	0.539
5 Days	0.33	[3.32]	0.21	[2.68]			0.181
5 Days	0.00	[0.00]	0.04	[0.55]	−73.71	[−3.43]	0.430
7 Days	0.30	[3.64]	0.21	[3.16]			0.161
7 Days	0.03	[0.42]	0.07	[1.13]	−61.63	[−3.38]	0.346

Table 3: Panel Regressions for Daily Forward Rate Returns

The (log) daily returns of the 1 week forward FX rate Δf_{it} (denominated in dollars per local currency and expressed in percentage points) is regressed on a constant, a time dummy D_t marking the event window, the time dummy interacted with the MSCI weight change $(w^n - w^o)_i$ of all stocks in currency i and the time dummy interacted with the risk contribution $[\Sigma(w^n - w^o)]_i$ of a currency to the arbitrage portfolio. Formally,

$$\Delta f_{it} = a + b \times D_t + c \times D_t \times q_i^{-1}(w^n - w^o)_i + d \times D_t \times [\Sigma(w^n - w^o)]_i + \mu_{it}, \qquad E(\mu_t \mu_t') = \Omega.$$

The time period covers 2 years of daily forward rate returns in 22 currencies for the period of July 1, 1998 to July 1, 2000 and the additional event window. We report results for event windows of 3, 5 and 7 trading days. Panel A reports results where the currency specific elasticity q_i is proxied by the relative market capitalization, i.e. the average old and new index weights $q_i = \frac{1}{2}(w^n + w^o)_i$. Panel B reports the the same regressions where the elasticity is proxied by the currency specific trading volume according to the BIS 2001 trianual market survey, i.e. $q_i = Vol_i^{FX}$. The constant coefficient estimate a is not reported. Panel corrected z-values are reported in parenthesis. The adjusted R^2 states the explanatory power for the event window period.

Event Window	b	[z]	c	[z]	d	[z]	Adj. R^2
Panel A: Capitalization Based Exchange Rate Elasticities (Forward Rates, N=22)							
3 Days	0.82	[4.63]	0.65	[3.03]			0.452
3 Days	0.36	[2.26]	0.46	[2.25]	−73.10	[−2.42]	0.592
5 Days	0.60	[4.34]	0.51	[3.07]			0.325
5 Days	0.16	[1.32]	0.33	[2.10]	−68.57	[−2.93]	0.500
7 Days	0.54	[4.49]	0.46	[3.29]			0.292
7 Days	0.20	[1.89]	0.32	[2.42]	−54.21	[−2.74]	0.414
Panel B: Volume Based Exchange Rate Elasticities (Forward Rates, N=22)							
3 Days	0.68	[4.29]	0.42	[3.43]			0.429
3 Days	0.24	[1.54]	0.26	[2.18]	−75.27	[−2.47]	0.578
5 Days	0.50	[4.05]	0.39	[3.96]			0.312
5 Days	0.09	[0.75]	0.23	[2.46]	−69.58	[−2.95]	0.493
7 Days	0.45	[4.32]	0.34	[4.38]			0.282
7 Days	0.13	[1.27]	0.23	[3.01]	−55.17	[−2.76]	0.408

Table 4: Cospectrum Within and Across Currency Groups

The mean and standard deviation of the cospectrum between currency returns is reported for four different spectral frequency bands B. In Panel A, currency pairs are drawn within groups $W+H+$ or $W-H-$, for which arbitrageurs are expected to trade in the same direction for both currencies. In Panel B, currency pairs are combined across groups where one currency is drawn from group $W+H+$ and the other from group $W-H-$. For the latter currency pairs arbitrage positions in opposite directions are expected, namely long and short positions, respectively. In Panel C, the sign of the cospectrum difference $\Delta Cosp(i,j,B)$ is reported for the 68 within group currency pairs and the 72 cross group currency pairs. The Fisher test reports the p-values for the null hypothesis that there is no association between the sign of the cospectrum change and the type of currency pair, namely within or cross group. The event period covers the 7 trading days from November 24 to December 4, 2000 and the control period the 7 trading days from September 8 to September 18. The Wilcoxon signed-rank test tests the null hypothesis that currency pairs have the same cospectrum during the event and control period. The very high frequency band aggregates currency comovements which occur within 15 minute intervals, the high frequency band corresponds to comovements from 15 minutes to 1 hour, the medium frequency band captures comovement from 1 hour to 4 hours and the low frequency band sums up the remaining low frequencies.

Panel A: Cospectrum for Currency Pairs within Groups $W+H+$ and $W-H-$, N=68

	Event Period		Control Period		Difference		Signed-Rank Test	
Frequency Band B	Mean	S.D.	Mean	S.D.	Mean	S.D.	Z-Value	P-Value
Very High	17.36	18.71	8.97	13.47	8.39	9.35	5.116	0.0000
High	12.45	13.65	12.81	19.86	−0.36	9.41	−0.375	0.7080
Medium	14.35	15.76	14.46	20.9	−0.11	8.50	0.140	0.8883
Low	5.29	8.29	8.29	15.27	−3.00	7.67	−1.358	0.1746

Panel B: Cospectrum for Currency Pairs across Groups $W+H+$ and $W-H-$, N=72

	Event Period		Control Period		Difference		Signed-Rank Test	
Frequency Band B	Mean	S.D.	Mean	S.D.	Mean	S.D.	Z-Value	P-Value
Very High	−0.75	2.01	1.16	3.12	−1.91	3.56	−3.642	0.0003
High	0.11	1.78	0.77	1.84	−0.66	2.51	−2.419	0.0156
Medium	0.03	1.50	−0.34	0.90	0.38	1.34	3.300	0.0010
Low	0.18	0.70	0.00	0.93	0.18	1.20	0.713	0.4760

Panel C: Sign of Cospectrum Difference $\Delta Cosp(i,j,B)$ for Within and Cross Group Currency Pairs

	Within Group Pairs			Cross Group Pairs			Fisher Test	
Frequency Band B	>0	≤ 0	All	>0	≤ 0	All	1-Sided	2-Sided
Very High	45	19	68	27	45	72	0.000	0.000
High	29	35	68	29	43	72	0.338	0.604
Medium	33	31	68	52	20	72	0.010	0.021
Low	33	31	68	39	33	72	0.448	0.864

Table 5: Spectral Band Regressions

The change in the cospectrum $\Delta Cosp(i,j,B)$ between the event period and the control period is calculated for different currency pairs (i,j) and four different spectral bands $B = VH, H, M, L$. The frequency bands aggregate changes in currency return comovements for periods less than 15 minutes (VH), from 15 minutes to 1 hour (H), from 1 hour to 4 hours (M) and the remaining low frequencies (L). The change in the cospectrum $\Delta Cosp(i,j,B)$ is explained by the quadratic form $\Delta\widehat{e}_i(c,d) \times \Delta\widehat{e}_j(c,d)$ in two parameters (c,d). Formally, we have non-linear regression

$$\Delta Cosp(i,j,B) = \Delta\widehat{e}_i(c,d) \times \Delta\widehat{e}_j(c,d) + \epsilon_{ijB} \qquad \text{for } B = VH, H, M, L,$$

where we define expected returns functions in currencies i and j as

$$\begin{aligned} \Delta\widehat{e}_i(c,d) &= c \times q_i^{-1}(w^n - w^o)_i + d \times [\Sigma(w^n - w^o)]_i \\ \Delta\widehat{e}_j(c,d) &= c \times q_j^{-1}(w^n - w^o)_j + d \times [\Sigma(w^n - w^o)]_j. \end{aligned}$$

Panels A and B use as elasticity parameter the average MSCI stock market capitalization, $q_i = \frac{1}{2}(w_i^n + w_i^o)$, and panels C and D the daily currency trading volume $q_i = Vol_i^{FX}$ according to the BIS 2001 trianual market survey. Panels A and C report the results for all currency pairs $(N = 595)$, and Panels B and D only for the most liquid currency pairs which have liquid forward markets $(N = 231)$.

Frequency Band (B)	c	[t]	d	[t]	F-Test	Adj. R^2
Panel A: Capitalization Based Exchange Rate Elasticities, All Currency Pairs, N=595						
Very High	1.72	[5.58]	−523.00	[−24.61]	76.16	0.203
High	1.38	[1.58]	−39.39	[−0.37]	0.32	0.002
Medium	0.00	[0.00]	0.00	[0.00]	0.00	0.000
Low	0.00	[0.00]	0.00	[0.00]	0.00	0.000
Panel B: Capitalization Based Exchange Rate Elasticities, Currency Pairs with Forward Rates, N=231						
Very High	2.94	[7.02]	−627.03	[−28.27]	108.42	0.482
High	2.26	[1.37]	−102.43	[−0.75]	0.25	0.000
Medium	0.00	[0.00]	0.00	[0.00]	0.00	0.000
Low	0.84	[0.29]	−27.72	[−0.11]	0.01	0.009
Panel C: Volume Based Exchange Rate Elasticities, All Currency Pairs, N=595						
Very High	1.28	[4.01]	−549.35	[−26.96]	91.50	0.233
High	1.26	[1.19]	−29.13	[−0.25]	0.18	0.000
Medium	0.90	[0.77]	−19.87	[−0.16]	0.07	0.000
Low	0.35	[0.16]	−7.78	[−0.03]	0.00	0.000
Panel D: Volume Based Exchange Rate Elasticities, Currency Pairs with Forward Rates, N=231						
Very High	1.60	[3.43]	−555.05	[−24.22]	78.02	0.400
High	2.80	[2.03]	−92.20	[−0.88]	0.58	0.000
Medium	2.31	[1.83]	−75.90	[−0.15]	0.47	0.000
Low	1.33	[0.72]	−29.18	[−0.20]	0.07	0.000

Figure 1: The percentage weight change $2(w^n - w^o)/(w^n + w^o)$ of each country in the MSCI global index is plotted against the log level of the old weight.

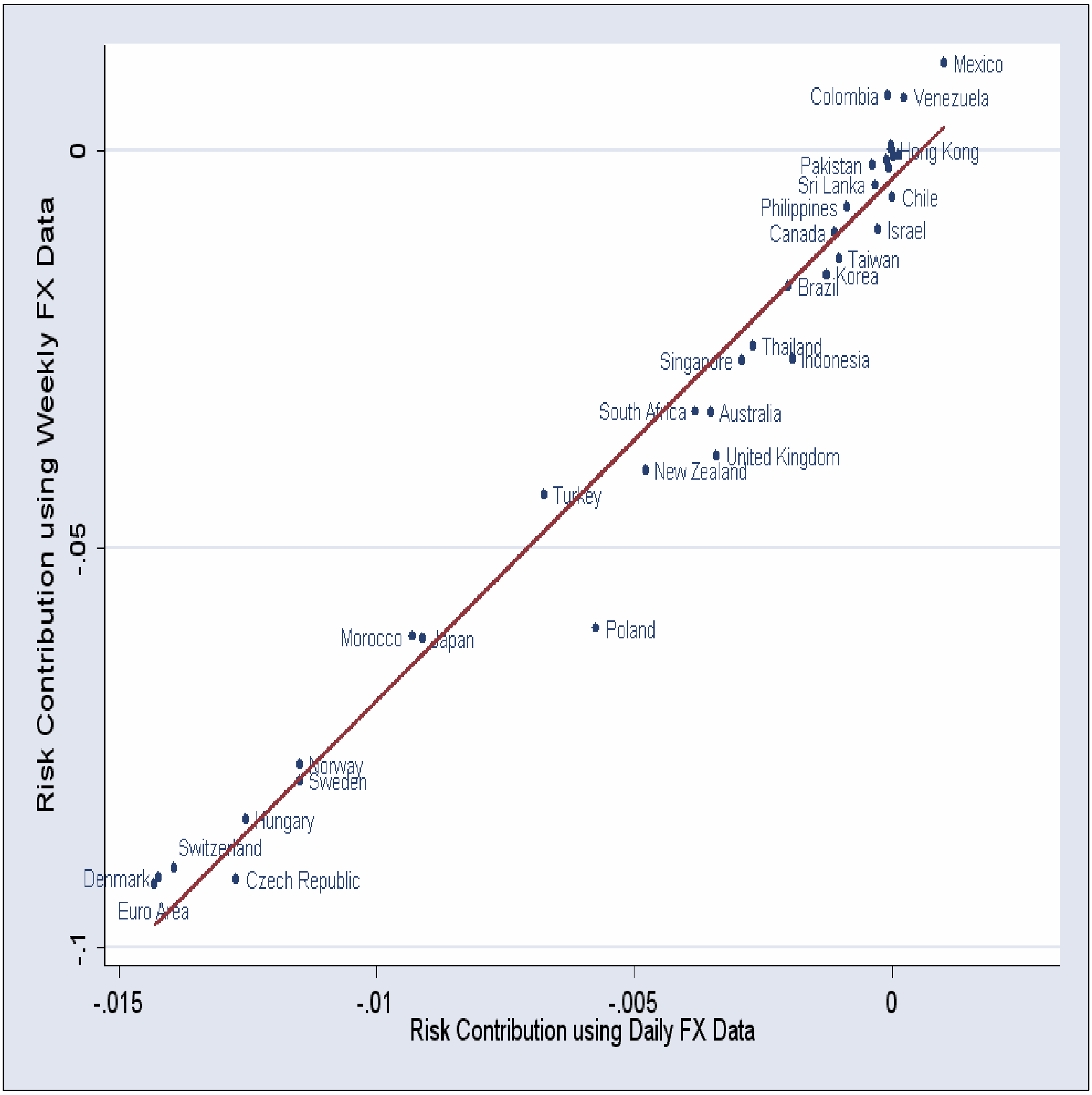

Figure 2: The arbitrage risk contribution $\Sigma(w^n - w^o)_i$ of each currency i is plotted using weekly FX data against the estimate resulting from using daily FX data. The correlation between the alternative measures is 0.984.

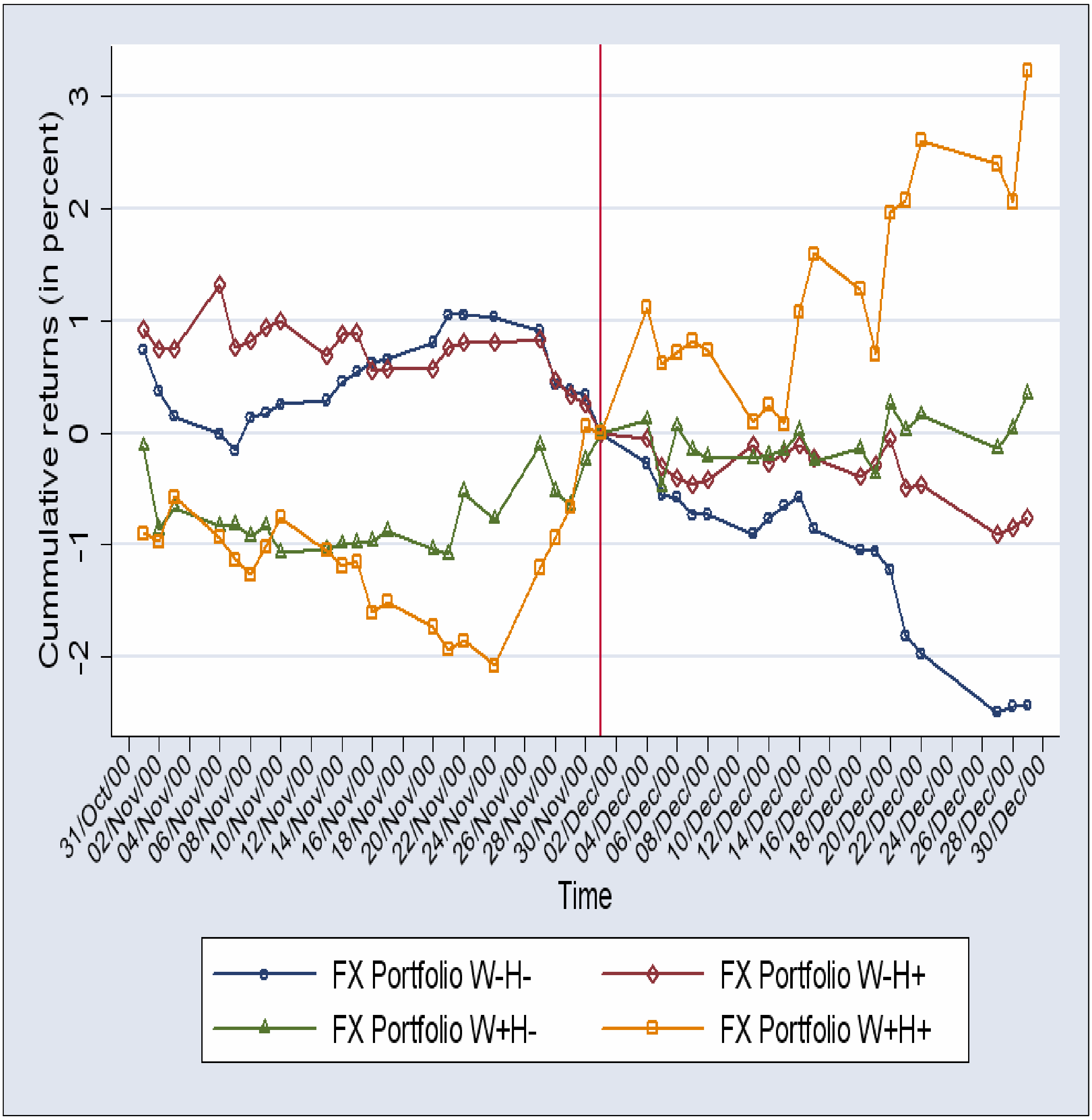

Figure 3: Currencies are sorted into those with above/below the median percentage weight change ($W+/W-$) and in a second sort into those with above/below median hedge value ($H+/H-$). All currencies in the sorted portfolio are equally weighted and their performance is plotted relative to an equally weighted currency portfolio composed of all 37 currencies.

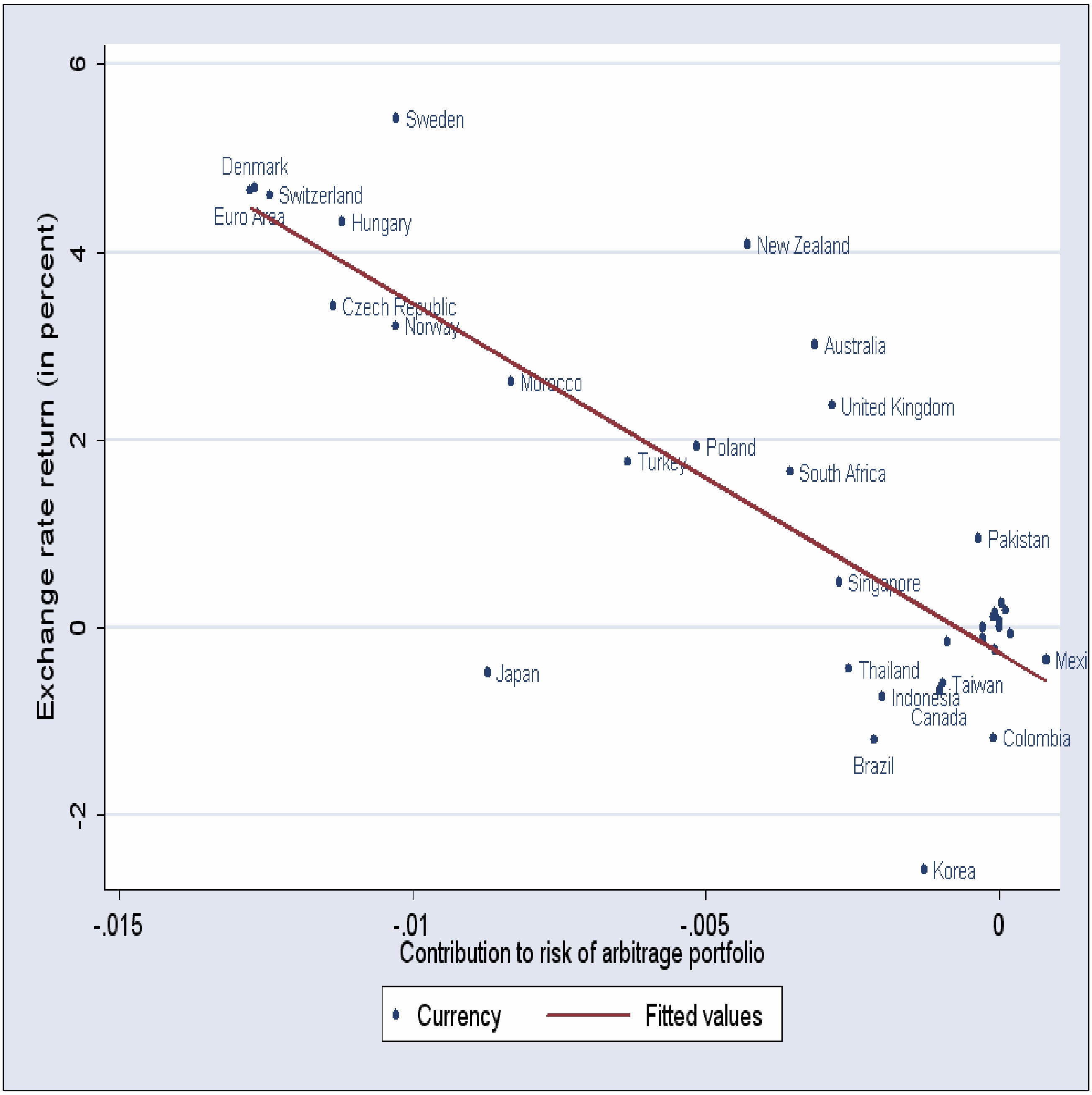

Figure 4: Plotted are exchange rate returns for 37 currencies over the 5 day event window against the arbitrage risk $\Sigma(w^n - w^o)_i$ in each currency.

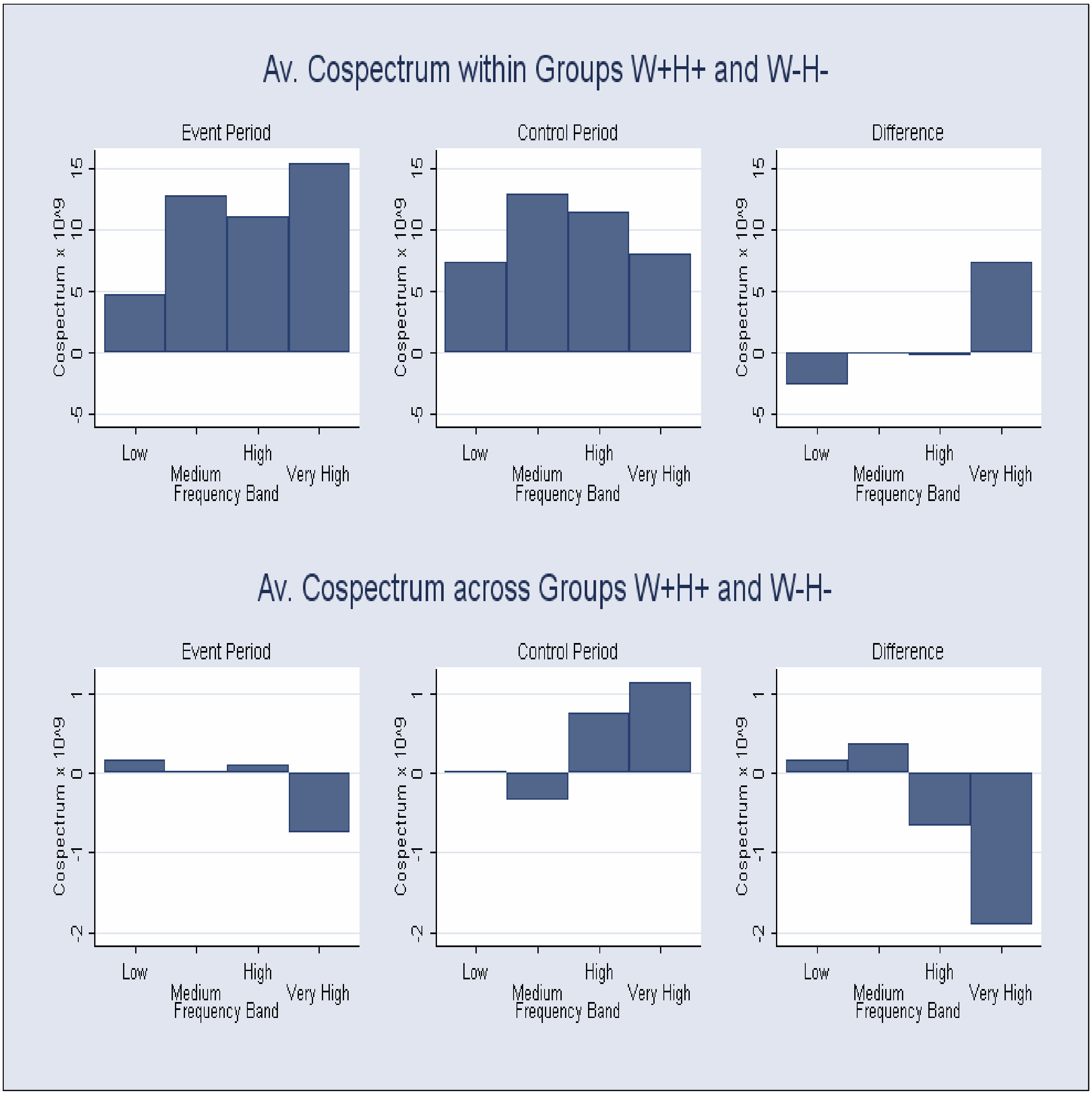

Figure 5: An average cospectrum is plotted for four different frequency bands B where (i) the currency pairs are drawn within the two groups $W+H+$ and $W-H-$ representing the most desirable and least desirable currencies and (ii) across groups where one currency is drawn from group $W+H+$ and the other from group $W-H-$. Column (1) graphs the average cospectrum $\overline{Cosp}(B)^{Event}$ over all pair permutations for the event period, column (2) the corresponding cospectrum $\overline{Cosp}(B)^{Control}$ for the control period and column (3) documents the change $\Delta\overline{Cosp}(B)$ in the cospectrum. The very high frequency band aggregates currency comovements which occur within 15 minute intervals, the high frequency band corresponds to comovements from 15 minutes to 1 hour, the medium frequency band captures comovements from 1 hour to 4 hours and the low frequency band sums up the remaining low frequencies.

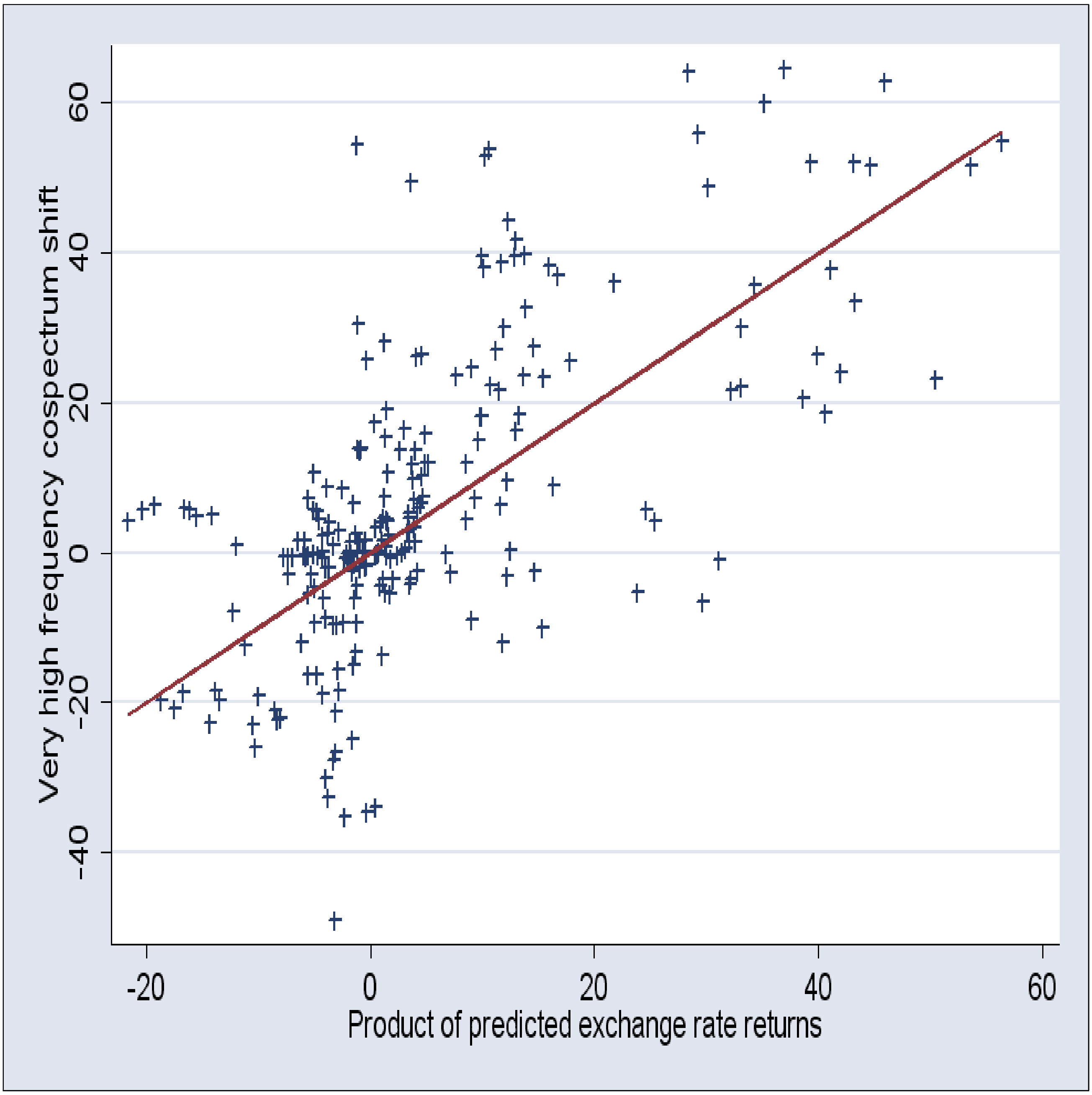

Figure 6: The shift of the very high frequency cospectrum $\Delta Cosp(i,j,VH)$ between all pairs formed by the 22 most liquid currencies is plotted against the product of the predicted event period return $\Delta\widehat{e}_i \times \Delta\widehat{e}_j$.